SELECT AMUSEMENTS

IN

PHILOSOPHY AND MATHEMATICS;

PROPER

For agreeably exercising the Minds of

YOUTH.

TRANSLATED FROM THE FRENCH OF M. L. DESPIAU,

Formerly Professor of Mathematics and Philosophy, at Paris:

WITH

SEVERAL CORRECTIONS AND ADDITIONS,

PARTICULARLY

A LARGE TABLE

OF THE

CHANCES OR ODDS AT PLAY.

The whole recommended as an useful Book for Schools; by

DR. HUTTON,

Professor of Mathematics, at Woolwich.

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SIR,

Woolwich, July 13, 1801.

IN answer to your request, to examine and give you an opinion of a work, entitled "Philosophical and Mathematical Amusements, &c," being a translation from the French of Despiau, with additions, and improvements, I have no hesitation in declaring my opinion, that it is a very curious and ingenious work, comprising a great deal of useful matter in a small compass, and well adapted for communicating the knowledge of a great variety of interesting particulars, in a manner at once familiar, clear, and amusing. To young persons especially, and in schools, this work must prove peculiarly useful; and the extensive tables of chances on all sorts of games, at the end of the work, must be singularly interesting to all such persons as attend or play at any kind of games, by exhibiting at one glance the chances or odds in a great variety of cases.

I am, Sir,

Your most humble Servant,

CHA. HUTTON.

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ERRATA.

Page 40, line 7, for *fiths* read fifths.
p. 71, l. 5, for *arraged* read arranged.
p. 93, l. 5, for *followigg* read following.
p. 111, l. 3, for *a certain* read an equal.
p. 143, in the second column, l. 13, of the table, insert a 6 before the 0.
p. 353, l. 10, from the bottom, for *Execuling* read Exciting.

MATHEMATICAL AND PHILOSOPHICAL AMUSEMENTS.

TO trace out the origin of amufements, it appears that it would be neceffary to go back to the earlieft ages of the world. For, mankind, being expofed to a variety of fatiguing labours, which exhauft both the mind and the body, have at all times exercifed their ingenuity in devifing means to difpel melancholy, and to revive the depreffed fpirits.

The remedies pointed out by nature for this purpofe, are reft, proper nourifhment, and cheerfulnefs: each day indeed exhibits in the fame individual a new being, in good or bad fpirits, according to the impreffions made on the animal œconomy by reft, a change of food, and various other circumftances.

The mind is too intimately connected with the body not to participate in the evils by which it is affected; but to the former, rest alone is not sufficient: to revive its powers, and to exhilarate the spirits by a proper stimulus, a change of objects, amusing conversation, agreeable news, and other things of the like kind, are necessary.

Every one knows that the spirits are depressed by too long application to gloomy or serious objects: to remedy this evil, others more amusing must be substituted in their stead; the least trifle or toy is often capable of giving to the mind the most tranquil and agreeable impressions; and during this state of peace and repose new spirits are created, which produce a change in the whole frame.

Walking, hunting, dancing, and music, are excellent sources of amusement; but they are not the only ones to which the necessity of unbending the mind, and filling up a vacant hour, have given birth.

The game of chess, it is said, had its origin at the siege of Troy; being invented by Palamedes to amuse the Grecian chiefs, disgusted with the tediousness of the siege.

Cards and tennis were invented by the Lydians, a people of Asia Minor, among whom, according to the antiquarians, all games had their origin: these people, it is well known, were so much addicted to voluptuousness and gaiety, that to express a thought-less, careless action, it was said, proverbially, to have been done *Lydio more.*

Amusements then are remedies invented to revive the depressed spirits, and to render the mind capable of resuming its usual labours with greater success; but a wise man will employ them with moderation, and will consider them as objects calculated to unbend the mind, and not to occupy it entirely.

Cicero told his son that amusements ought to be employed like sleep; which, if used to excess, becomes dangerous, and instead of reviving the powers of the mind, renders them torpid.

On this subject Cassian relates an expression of the Apostle John, which deserves to be recorded: —A hunter, who one day saw him caressing a partridge, seemed astonished that so pious a man should amuse himself with such a trifling object. "My good friend," said the Apostle, "what have you got in your hand?" "A bow," replied the hunter; "and why is it not bent?" added the Apostle. "If it were always bent," returned the hunter, "it would lose it's strength." "Be not then surprised;" continued the Apostle, "that the mind also should sometimes require relaxation."

Sidronius Hoschius, the Flemish Ovid, has expressed the same thought, with great elegance, in the following lines:—

Deficiet sensim qui semper tenditur arcus;
Ferre negat segetes irrequietus ager.

The latter comparison has been employed by Seneca, who ſays,—" the mind of man is like thoſe fields, the fertility of which depends on their being allowed certain periods of reſt, at the proper ſeaſons." This philoſopher had remarked, that too long and too aſſiduous labour exhauſts the mind, throwing it into a kind of languor; but, that by relaxation it is revived, and rendered fitter for reſuming its occupations.

How often are people difficulted by problems merely of an amuſing nature, the whole ſolution of which depends upon ſome elementary calculation, the natural properties of certain bodies, or mathematical combinations! We admire the ſagacity and pretended knowledge of the perſon who propoſes them; and yet nothing is eaſier than to comprehend and even to execute what thus excites our aſtoniſhment and wonder. Why then ſhould not we acquire the knowledge neceſſary to enable us to propoſe problems and enigmas ourſelves?

Intricate and puzzling queſtions have, at all times, formed a part of the amuſements of the moſt poliſhed nations; and they have been received with avidity even by young perſons when preſented under the agreeable form of an enigma or recreation. I will even venture to aſſert, if we are allowed to judge of others by what we experience ourſelves, that we are ſometimes conducted to the higher parts of the moſt abſtract ſtudies, by the flowery path of ſome experiment, which we at firſt conſidered as an object of mere curioſity.

It is well known that the high reputation of Solomon induced the queen of Sheba to come from the remoteſt part of Ethiopia, to admire the wiſdom of that great prince—the wonder of his age. She came, ſays the ſcripture, to try his wiſdom, by propoſing to him enigmas. Solomon ſatisfied her on every point, and anſwered all her queſtions with ſo much propriety, that the queen returned in the utmoſt joy, unable to contain the tranſports of her admiration, excited by the wiſdom and magnificence of that great king.

The celebrated Æſop became the favourite of Crœſus, merely on account of his ingenious fables, which contained the moſt refined morality, and inſtructions the more delicate, as they conveyed cenſure without wounding that ſelf-love which is ſo natural to man. Fable, in the hands of this great genius, ſeemed a rod dipped with ſo much art in the gall of ſatire, as to have none of its bitterneſs or ſeverity. Nathan repreſented to David the enormity and injuſtice of the crime he had committed, in regard to Uriah, only under the veil of an iningenious allegory; which produced a greater and ſpedier effect on the mind of the Monarch than if the prophet, arming himſelf with the thunder of his eloquence, had pointed out to him, in a direct manner, the horror of his offence.

Does not Solomon desire the sluggard and the spendthrift to consider the ways of the ant; and does not Æsop seem to have borrowed from this idea his fable of the ant and the grasshopper? That great prince, in delineating the portrait of true wisdom, paints her in saying, " To understand a parable, and the interpretation, the words of the wise, and their dark sayings."

Mental amusements then have been esteemed, in all ages, and by persons of every condition; and the pleasure they excite is the purer as they affect only the more delicate parts of the mind. The human intellect, as is well known, has its peculiar pleasures: every thing that increases knowledge, pleases and exalts it; we are always gratified when we comprehend a difficulty which has checked the progress of others, or have unveiled a mystery, concealed from persons possessed of less penetration than ourselves.

Besides, these amusements, purely intellectual, may be enjoyed at little expence; they do not fatigue the body, on which they make no impression; and, on this account, they ought to be preferred to sensual pleasures, the enjoyment of which creates disgust, injures the health as well as fortune, and almost always deranges the economy of a peaceful and tranquil life.

The claſs of Mathematicians has always arrogated the right of treating of mathematical and philoſophical recreations. In compiling the preſent collection, Ozanam, and thoſe who have written on the ſame ſubject, have been my guides; and from their works I have ſelected the greater part of what I now offer to the public; for theſe amuſements are the production neither of one man, nor one age, but of a great number of the learned, of artiſts, and of many ages of reſearch and of obſervation.

By the long experience I have had, I am induced to hope that young perſons, who are often diſguſted with the formality of ſtudy, and who, on that account, ſometimes conceive an averſion to the moſt uſeful branches of ſcience, will find in the greater part of the amuſements which are here preſented to them, ſome things ſuited to their taſte, and eaſy to be comprehended. When the firſt difficulties are ſurmounted, they will become ſo many ſteps to conduct them gradually, by the moſt agreeable path, to the ſolution of problems, which at firſt may appear too difficult and abſtruſe for their age.

As it is impoſſible to underſtand properly all theſe amuſements without the knowledge of certain principles, the application of which is often neceſſary, they are preceded by an introduction, calculated to facilitate the ſolution of the moſt difficult problems.

*** The reader is requested to observe that the figures, inclosed within parenthesis, which occur in the course of the following work, refer to that section in the introduction, where the necessary explanation will be found.

INTRODUCTION.

1. DIFFERENT symbols or signs, established by general practice, are sometimes employed in order to simplify calculations, and facilitate the resolution of certain problems. Thus,

$+$ signifies plus or more
$-$ .... minus or less
$=$ .... equal
$>$ .... greater
$<$ .... less
$\times$ .... multiplied by
$\div$ .... divided by.

Thus, it may be easily conceived that $2 + 3 = 5$; that $3 - 2 = 1$; that $3 > 2$; that $2 < 3$; and that $12 \div 3$ or $\frac{12}{3} = 4$.

Of Fractions.

2. Befides the application of the common rules to whole numbers, with which every body is acquainted, it is fometimes indifpenfably neceffary to perform the fame operations with fractional numbers.

A Fraction is one or more parts of a whole. Every fraction is expreffed by two characters, placed one above the other, with a line between them, in this manner: $\frac{3}{4}$, $\frac{a}{b}$, &c. The upper character, which is called the numerator, expreffes how many parts are taken of the whole; and the other, called the denominator, denotes the quality of thefe parts. Thus the fraction $\frac{3}{4}$ fignifies that the whole is divided into fourths, and that 3 of them are taken.

It hence follows, that a fraction is greater according as the numerator is greater; and, on the other hand, lefs as the denominator is greater. Thus $\frac{5}{7} > \frac{4}{7}$; $\frac{3}{8} < \frac{3}{5}$. By a neceffary confequence all fractions, the two characters of which are equal, denote exactly the fame value: $\frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6}$ &c.

3. To reduce a whole number to a fraction, which fhall have a determinate denominator, we muft multiply the whole number by the given denominator, and place the product above the latter. Thus 8, reduced to a fraction, having 3 for its denominator,

is $\frac{24}{7}$; and 5 reduced to a fraction having the same denominator as $\frac{2}{7}$, is $\frac{35}{7}$.

4. To reduce two fractions to the same denominator, the numerator of the first must be multiplied by the denominator of the second, and the numerator of the second by the denominator of the first: these two products will be the numerators of the two new fractions; and the product of the two denominators will be the common denominator. Thus $\frac{2}{3}$ and $\frac{3}{5}$, reduced to the same denominator, give $\frac{10}{15}$ and $\frac{9}{15}$. Any number of fractions may, in like manner, be reduced to a common denominator, provided that each numerator be multiplied by the denominators of the other fractions, and that the product of all the denominators be taken for a common denominator. Thus, for example, the three fractions $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{7}$, when reduced to the same denominator, give $\frac{21,\ 56,\ 36}{84}$.

5. To add two fractions, we must first reduce them to a common denominator, and then add their numerators. Thus the sum of the two fractions $\frac{3}{5}$ and $\frac{4}{7}$, is $\frac{21+20}{35} = \frac{41}{35}$.

6. To subtract one fraction from another, they must first be reduced to the same denominator, and the numerator of the less must then be taken from the numerator of the greater. Hence the difference of the fractions $\frac{3}{7}$ and $\frac{2}{5}$ is $\frac{1}{35}$.

7. To multiply two fractions together, we must make a new fraction, the numerator of which shall be the product of the two numerators, and the denominator the product of the denominators. Thus the product of $\frac{2}{3}$ by $\frac{3}{4}$ is $\frac{6}{12}$.

8. To divide one fraction by another, we must make a fraction, the numerator of which shall be equal to the product of the numerator of the first multiplied by the denominator of the second; and the denominator equal to the product of the numerator of the second multiplied by the denominator of the first. The quotient of $\frac{3}{5}$ divided by $\frac{1}{3}$ will therefore be $\frac{9}{5}$.

9. Sometimes it is necessary to simplify a fraction, by reducing it to its simplest expression, or what is called its lowest terms: nothing is necessary for this purpose, but to divide the numerator and denominator by the greatest common measure or divisor. Thus the fractions $\frac{3}{6}$ and $\frac{4}{20}$, reduced to their simplest expression, give $\frac{1}{2}$ and $\frac{1}{5}$.

Of Powers.

10. By the *power* of a quantity, is understood its product by unity or by itself a certain number of times. Thus, the first power of 2 is 2; its second power or square is 2 × 2; its third power or cube, is 2 × 2 × 2, and so on. Hence it is evident that, to obtain any power whatever of a given quantity, it must be multiplied by itself as many times less 1, as are equal to the number which denotes that power.

The power of any quantity is expreffed fometimes algebraically, or numerically, by the figure which denotes its degree, as in the following examples: a^1, a^2, a^3, a^4, &c. 4^1, 4^2, 4^3, 4^4, &c.

11. An algebraic character is fometimes accompanied by two figures, as $2b^3$. The firft of thefe is called the *coefficient*, and the fecond the *exponent*: the former denotes how many times the quantity is added to itfelf; and the fecond indicates the power. Thus, the value of b, being fuppofed equal to 3, we fhall have $2b^3 = 54$.

12. By the *root* of a quantity, is meant a number which being multiplied one or more times by itfelf, will give that quantity. There is therefore a *fquare root*, *cube root*, &c.

The different roots of quantities may be expreffed by the following figns: $\sqrt[2]{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, in this manner $\sqrt[2]{a}$, $\sqrt[3]{a}$, $\sqrt[4]{a}$, $\sqrt[2]{16}$, $\sqrt[3]{27}$.

13. We fhall now fhew how to extract the fquare root of any quantity; that is to fay, how to find a number, which being multiplied by itfelf, will give that quantity, if it be a complete fquare, or at leaft the greateft fquare which it contains.

EXAMPLE I.

Let the number, the fquare root of which is required, be 1156. Firft divide this number from right to left into periods of two figures, and then proceed as follows.

Find the greateſt ſquare contained in 11, which is 9, and write down its root 3, as ſeen in the annexed example. Square 3, which gives 9, ſubtract it from 11, and the remainder will be 2. Then bring down the next period, which is 56, that it may ſerve as a dividend along with the figure 2 on its left. Take 6 as a diviſor, that is to ſay, the double of the root 3 already found; place it on the left, and find how often it is contained in 25; the quotient will be 4, which muſt be written down in the root after 3; and alſo after 6, the diviſor, which will give 64. Then multiply the laſt number by the ſecond root 4, and the product will be 256. As there is no remainder, it is a proof that 1156 is a perfect ſquare, the root of which is 34. Had the laſt product been too large to be ſubtracted, it would have been neceſſary to diminiſh the laſt figure in the root, in order to make it ſmall enough for that purpoſe.

```
     11, 56 ( 34
      9
64 |  256
 4 |  256
```

EXAMPLE II.

What is the Square Root of the Number 214369?

As in the preceding example, we muſt firſt divide the number into periods of two, from right to left, and there will be as many figures in the root as there are periods.

Then as the greateſt ſquare contained in 21 is 16 the ſquare root of which is 4, write down the 4 in,

```
       21, 43, 69 ( 463
       16
86)    543
6      516
923)   2769
3      2769
```

the root; ſquare 4, which will give 16, and having ſubtracted it from 21, the remainder will be 5. Bring down the following period 43, which with the preceding figure muſt be divided by the double of the root already found, that is to ſay, by 8. The quotient of 54 divided by 8 is 6, which muſt be placed after the firſt root 4, and alſo after 8 the diviſor; then multiply 86 by 6, and ſubtract the product 516 from 543. Place the remainder 27 under 516, and bring down the next period 69. Take, as the diviſor of 276, the double of the two roots already found, which is 92. Divide 27 by 9, and place the quotient 3 in the root after 46, and alſo after 92. Then multiply 923 by 3, and if the product 2769 be ſubtracted from the number 2769, there will be no remainder. The truth of this operation may be proved by ſquaring 463; that is to ſay, by multiplying it by itſelf.

After the laſt ſubtraction, if any thing remains, it is a proof that, though the root found is not exactly the real root, it does not want unity to be ſo; but if it were required to approach ſtill nearer to the real root, nothing would be neceſſary but to reduce the remainder to decimals, and to continue the operation, taking care to ſeparate the whole numbers in the root from the decimals. However, as we propose here only to give a few amusing problems, there will be no necessity for carrying the extraction of the square root beyond whole numbers.

Of Equations.

14. As certain questions cannot be easily resolved without some knowledge of analysis and equations, we shall here give a short explanation of them, and such as may be easily understood.

By *equations* is meant the application of numerical and algebraic rules to the solution of different questions, which may be proposed respecting quantity.

The first and most difficult thing in analysis, is to comprehend properly the state of the question, and the relation which the known quantities bear to the unknown, in order that they may be clearly expressed in an equation.

Every equation is composed of two members, separated by the sign $=$; and each member may consist of several terms. An example of the whole may be seen in the following equations:

$7=3+4$; $8-5=2+1$; $3\times4=12$; $\frac{9}{3}=3$.

There may be equations also consisting of algebraic quantities alone, or in which arithmetical quantities are mixed with algebraic ones, as in the following:

$x+b=a$; $x-y=a+b$; $3a-b=4c-2x$.

General Rules in regard to Equations.

RULE I.

15. Any quantity may be tranſpoſed from one member of an equation to another, without deranging the equation, provided that the ſigns be changed.

Thus, as $12 - 3 = 9$, we may write $12 = 9 + 3$. For the ſame reaſon if $a - x + 3b = d - y$, we ſhall have $a + y - d = x - 3b$.

This method of operation, in regard to equations, is called *tranſpoſition*, and is employed when it is neceſſary to free one member of an equation from any quantity connected with it either by addition or ſubtraction.

RULE II.

16. When an unknown quantity is involved in an equation either by multiplication or diviſion, it may be diſengaged from it, in the firſt caſe, by diviſion; and in the ſecond by multiplication.

For example, if $3x = b$, then $x = \frac{b}{3}$; and if $\frac{y}{3} = a$, then $y = 3a$.

This method of diſengaging an unknown quantity will be more eaſily comprehended, if we give determinate values to the quantities a and b. If we ſuppoſe, for example, that $b = 12$, and $a = 8$, the two equations above mentioned will be reduced to the following, $x = \frac{12}{3} = 4$, $y = 24$.

17. It appears therefore, that the whole art of analysis consists, first, in comparing in the equations the unknown with the known quantities, and disengaging them from each other by the means already pointed out, in such a manner, that the known quantity may remain alone in one member of the equation, and the unknown in the other.

To facilitate the solution of algebraic questions, the unknown quantities are generally denoted by some of the last letters of the alphabet, x, y, z; and the known quantities by some of the first, as a, b, c, &c.

Of Ratios and Proportions.

18. *Relation or Ratio* is what results from the comparison of two quantities. As two quantities may be compared with each other two ways, ratio is distinguished into two kinds, arithmetical and geometrical.

Arithmetical relation, is that of two quantities compared with each other by subtraction.

Geometrical relation, is that of two quantities compared with each other by division.

Thus, for example, the arithmetical ratio of 12 to 4 is 8; and the geometrical ratio of the same quantities, is 3; for $12 - 4 = 8$, and $\frac{12}{4} = 3$.

19. *Proportion* is an equality of ratios. As there are two kinds of ratio, there are also two kinds of proportion, arithmetical and geometrical: the first consists in an equality of differences, and the second in an equality of quotients.

Every ratio is expressed by two terms; the first of which is called the *antecedent*, and the second the *consequent*.

Two equal ratios form a proportion; which is either arithmetical or geometrical, according as they contain either the same difference or the same quotient. Thus $3 . 5 \because 7 . 9$, expresses an arithmetical proportion; the meaning of which is, that 3 is arithmetically to 5, as 7 is to 9; and $6 : 3 :: 16 : 8$, expresses a geometrical proportion; the meaning of which is, that 6 is geometrically to 3, as 16 is to 8.

The first and last terms of each of these proportions are called the *extremes*; and the other two, the *means*.

20. Proportion is *continued* when the same term is the consequent of that which precedes it, and the antecedent of that which follows it. Thus the two following proportions, one of which is arithmetical, and the other geometrical, are continued, viz. $\div 3 . 5 . 7$; $∺ 4 : 8 : 16$. The meaning of which is, $3 . 5 \because 5 . 7$; $4 : 8 :: 8 : 16$.

When a continued proportion has more than three terms, it is called a *progression*. Thus $\div 1 . 3 . 5 . 7 . 9$ is an arithmetical progression, and $∺ 4 : 8 : 16 : 32 : 64$ is a geometrical progression.

Properties of arithmetical Proportion and Progression.

THEOREM I.

21. In every arithmetical proportion, the sum of he extremes is equal to that of the means.

If $3 \cdot 5 \because 7 \cdot 9$, or $9 \cdot 6 \because 8 \cdot 5$,

Then $3+9=5+7$, and $9+5=6+8$.

It hence follows, that when three terms of an arithmetical proportion are known, we may easily find the fourth; for if the unknown term be an extreme, it will be found by subtracting the other extreme from the sum of the means; and if it be one of the means, by subtracting the other mean from the sum of the extremes.

If $a \cdot b \because c \cdot x$, or if $4 \cdot x \because 3 \cdot 8$,

$$\text{Then} \begin{cases} b+c=a+x \\ b+c-a=x \end{cases} \text{and} \begin{cases} 4+8=x+3 \\ 4+8-3=x \end{cases} \quad (15)$$

It hence follows also, that if two terms, as a and b, are given, a third arithmetical proportional to them may be easily found, in order to form an arithmetical progression. For if we suppose the required term to be x, we shall have:

$$\because a \cdot b \cdot x$$

$$\text{Then} \begin{cases} a \cdot b \because b \cdot x \\ a+x=b+b=2b \quad (20) \\ x=2b-a. \quad (14) \end{cases}$$

Consequently, to find a third arithmetical proportional to two given terms, we must subtract the first from double the second. Thus, the third arithmetical proportional to 3 and 7, will be $14-3=11$; and indeed $\because 3 \cdot 7 \cdot 11$.

An arithmetical mean proportional between two given terms, such as a and b, may be found with equal eafe; for if the required mean be denoted by x, we ſhall have

$$\because a \,.\, x \,.\, b \quad (20)$$

$$\text{Then} \begin{cases} a + b = 2x \\ \dfrac{a+b}{2} = x \quad (15) \end{cases}$$

Which indicates, that an arithmetical mean proportional to two quantities, is equal to the half of theſe quantities. Thus, the mean proportional between 9 and 13, is 11; for $\because 9 \,.\, 11 \,.\, 13$.

THEOREM II.

22. In every even arithmetical progreſſion, the ſum of all the terms, equally diſtant from the extremes, taken two and two, is equal to that of the extremes; and if it be odd, the ſum of the extremes, or of any two terms equally diſtant from the extremes, is the double of the mean term.

CASE I.

$$\text{If} \div 3 \,.\, 5 \,.\, 7 \,.\, 9 : 11 \,.\, 13,$$

$$\text{Then} \begin{cases} 5 + 11 = 3 + 13 \\ 7 + 9 = 3 + 13 \end{cases}$$

CASE II.

$$\text{If} \div 2 \,.\, 4 \,.\, 6 \,.\, 8 \,.\, 10,$$

$$\text{Then} \begin{cases} 2 + 10 = 2 \times 6 \\ 4 + 8 = 2 \times 6 \end{cases}$$

In the firſt caſe, the ſum of an arithmetical progreſſion, is equal to the product of the ſum of the extremes multiplied by half the number of terms; and in the ſecond, to the product of the mean multiplied by the number of terms.

THEOREM III.

23. In every arithmetical progreſſion, any term whatever is equal to the firſt and as many times the common difference as there are terms before it.

If ∴ 2 . 5 . 8 . 11 . 14.

$$\text{Then}\begin{cases} 14 = 2 + \overline{3 \times 4} \\ 11 = 2 + \overline{3 \times 3} \\ 8 = 2 + \overline{3 \times 2} \end{cases}$$

It hence follows, that we may eaſily find the value of any term of an arithmetical progreſſion, the firſt term, the common difference and the number of terms of which are known.

For example, the 121ſt term of an arithmetical progreſſion, the firſt term of which is 5 and the common difference 3, will be 365; for $5 + \overline{3 \times 120} = 365$.

Properties of Geometrical Proportion and Progreſſion.

THEOREM I.

24. In every geometrical proportion, the product of the extremes is equal to that of the means.

If 3 : 6 :: 4 : 8

Then $3 \times 8 = 6 \times 4$

Consequently, the fourth term of a geometrical proportion, the other three of which are known, may be easily found; for if the required term be an extreme, it will be equal to the product of the means divided by the other extreme; and if it be a mean, it will be equal to the product of the extremes divided by the other mean.

If $2 : 4 :: 3 : x$

$$\text{Then} \begin{cases} 2x = 4 \times 3 & (13) \\ x = \dfrac{4 \times 3}{2} = 6 \end{cases}$$

If $2 : y :: 3 : 6$

$$\text{Then} \begin{cases} 2 \times 6 = 3y & (15) \\ \dfrac{2 \times 6}{3} = y = 4 \end{cases}$$

Two terms being given, a third, geometrically proportional to them, may be easily found, in order to form a geometrical progression. Let us suppose that a third proportional is required to the terms a and b, and that the term sought is denoted by y. We shall then have

$$\div\!\div\ a : b : y$$

$$\text{Then} \begin{cases} a : b :: b : y \\ ay = b^2 \\ y = \dfrac{b^2}{a} \end{cases}$$

Consequently, to find a third term, geometrically proportional, we must divide the square of the second, or its product by itself, by the first term. Thus the third geometrical proportional to 3 and 6, will be $\dfrac{6 \times 6}{3} = 12$; and indeed $\div\!\div\ 3 : 6 : 12$.

A mean geometrical proportional between two terms, as a and b, may be found with equal eafe; for if this term be called x, we fhall have:

$$\div a : x : b$$

$$\text{Then} \begin{cases} xx \text{ or } x^2 = ab \\ \sqrt{xx} = \sqrt{ab} \\ x = \sqrt{ab} \end{cases}$$

Thus, if we fuppofe $a = 2$, and $b = 8$; x will be equal to the fquare root of 16, which is 4. And indeed $\div 2 : 4 : 8$.

THEOREM II.

25. Any term whatever of a geometrical progreffion is equal to the product of the firft term multiplied by the common ratio, raifed to that power the exponent of which is equal to the number of terms before it.

Let the geometrical progreffion be $\div 2 : 4 : 8 : 16 : 32 :$ &c.

The fifth term 32, for example, is equal to the product of 2, the firft, multiplied by 16, which is the fourth power of the ratio 2.

THEOREM III.

26. In every geometrical progreffion, the fecond term, lefs the firft, is to the firft, as the laft, lefs the firft, is to the fum of all thofe which precede it.

If $\div 2 : 4 : 8 : 16 : 32 :$ &c.

Then $4 - 2 : 2 :: 32 - 2 : 2 + 4 + 8 + 16$.

Rule of Three.

27. The *Rule of Three*, is an operation by which, when three terms of a geometrical proportion are known, a fourth, not known, may be found; and it is called *direct* when the similar terms increase in the same ratio. For example; if four men perform six yards of work in a certain time, it is evident that a greater number must perform more in the same time. On the other hand, if the similar terms, instead of increasing in the same ratio, must decrease, the rule is called *inverse*, as is the case in the following example: if four men perform a certain work in eight days, a greater number of men must perform it in a time proportionally less.

The rule of three direct, and the rule of three inverse, may be expressed by the following formulæ:

Men		Men		Yards		Yards
3	:	6	::	4	:	x
Men		Men		Days		Days
3	:	6	::	x	:	8

The rule of three is compound or simple, according as the terms are compound or simple. For example, the above two formulæ express each the simple rule of three. But if it were required to divide the profits of a commercial company among several partners, who have advanced certain capitals for different periods of time, it would be neceſſary to multiply the capital of each partner by the time, which would render the rule the compound rule of three.

As the rule of three is only the application of the formulæ of Theorem 1ſt (23), it is needleſs to enlarge farther on this ſubject.

ARITHMETIC.

ARITHMETIC and Geometry, according to Plato, are the two wings of the mathematician; and, indeed, the object of all mathematical questions, is to determine the ratios of numbers or of magnitudes. It may even be said, to continue the comparison of the ancient philosopher, that arithmetic is the mathematicians right wing; for, it is certain, that geometrical determinations would often afford very little satisfaction to the mind, if the ratios, thus determined, could not be reduced to numerical ratios. This justifies the common practice of beginning with arithmetic.

This science presents a wide field for speculation and curious research; but in the present selection, we shall confine ourselves to such things as are best calculated to excite the curiosity of those who have a taste for the mathematics, and who seek for recreations that may enable them to resume their more serious studies with greater success.

Of our Numerical Syſtem, and the different kinds of Arithmetic.

It has been generally obſerved, that all the nations with which we are acquainted, reckon by periods of ten; that is to ſay, after having counted the units, as far as ten, they begin again by adding units to a ten; when they attain to 20, they add units as far as 30, or three tens, and ſo on in ſucceſſion, till they come to 100, or ten tens; of ten times a hundred they form a thouſand, &c. Did this ariſe from neceſſity; was it occaſioned by any phyſical cauſe; or was it merely the effect of chance?

No one, after the leaſt reflection, will be inclined to aſcribe it to chance. It is not only probable, but might almoſt be proved, that this ſyſtem derives its origin from our phyſical conformation. All men have ten fingers, a very few excepted, who by ſome *luſus naturæ* have twelve. The firſt men began to reckon on their fingers. After having exhauſted them by counting the units, it was neceſſary for them to begin and to count them again, till they had exhauſted them a ſecond time; then a third time, and ſo on. Hence the origin of tens; which, being confined to the fingers, could not be carried beyond the number of ten without forming a new total, called a hundred; then another, conſiſting of ten hundreds, called a thouſand, &c.

A curious conſequence hence follows; which is, that if nature, inſtead of 10 fingers, had given us

12, our numerical syſtem would have been different. After 10, inſtead of ſaying ten plus one or eleven, we ſhould have aſcended by ſimple denominations to twelve, and ſhould have then counted twelve plus one, twelve plus two, &c. as far as two dozens. Our hundred would have been twelve dozens, a thouſand twelve times twelve dozens, &c.

A ſix fingered people, in all probability, would have had an arithmetic of this kind, which indeed would not have been inferior to that now in uſe, or rather would have been attended with ſome advantages, which our preſent numerical ſyſtem does not poſſeſs.

This method of numeration would have been as expeditious, and even more ſo than that now univerſally received. The number of characters, which would have been encreaſed only by two, to expreſs ten and eleven, would have been as little burthenſome to the memory, as the preſent characters; and this ſyſtem would have poſſeſſed ſome advantages, which ought to give us reaſon to regret that it was not originally adopted.

This, however, would no doubt have been the caſe, had philoſophy preſided when the ſyſtem was firſt formed; as it would have readily been ſeen that 12, of all the numbers almoſt between 1 and 20, poſſeſſes the advantage of being ſmall, and of having the greateſt number of diviſions, viz. 2, 3, 4, and 6, by which it can be divided without a remainder Befides, in this fyftem the periods of numeration would have had the advantage of being divifible, the firft from one to twelve, by 2, 3, 4, 6; the fecond, from one to a hundred and forty-four, by 2, 3, 4, 6, 8, 9, 12, 16, 24, 36, 48, 72; whereas, in our fyftem, the firft period from one to ten has only two divifors, 2 and 5; the fecond from one to a hundred has only feven, viz. 2, 4, 5, 10, 20, 25, 50, confequently fractions would have lefs frequently occurred in numerical operations.

But what would have been particularly convenient in this mode of numeration, is, that it would have introduced into all meafures the duodecimal divifions and fubdivifions. Thus, as the foot is divided into 12 inches, the inch into 12 lines, and the line into twelve points; the pound, in the like manner, would have been divided into 12 ounces, the ounce into 12 drams, the dram into 12 grains, or other denominations at pleafure; the day would have been divided into 12 portions, called hours; the hour into 12 others, which would have been equal to 10 minutes; and each of thefe into 12 inferior parts; and fo on in fucceffion.

Should it be afked, what would have been the advantage of this divifion, we might reply as follows: It is well known that when it is neceffary to divide any meafure into 3, 4, or 6 parts, a whole number of meafures of the lower denomination cannot always be found, or are found only by chance. Thus, the third or tenth of a pound avoirdupois does not always give an exact number of ounces; and the third of a pound sterling does not give an exact number of shillings. The case is the same with the bushel, and the greater part of other measures. These inconveniences, which render calculations complex, would not have occurred, had the duodecimal progression been universally adopted.

Stevin, a Dutch mathematician, proposed to adapt the divisions and subdivisions of all measures to the system of numeration since adopted by the French, making them to decrease in decimal progression. Thus, the fathom would have contained 10 feet, the foot 10 inches, the inch 10 lines, &c. This method, however, though attended with some advantages, is less perfect than the duodecimal, as it gives rise to a greater number of fractions.

A great many systems of arithmetic have been proposed, such as the binary, ternary, quaternary, &c; and even the duodenary; but there is no great reason to believe that any of them will ever be admitted into practice.

But we shall add nothing farther on the subject; for, as useful recreations are the object of this collection, we must exclude from it every thing too complex, or of too frivolous a nature.

We ſuppoſe that the reader is ſufficiently well acquainted with arithmetic, both in whole and fractional numbers, (2), to be able to comprehend every thing that relates to it.

Of ſome Properties of Numbers.

Under this head, we do not comprehend thoſe properties of numbers which engaged ſo much the attention of the ancients, and to which they aſcribed ſo many myſterious virtues. Every one, whoſe mind is not tinctured with credulity, muſt laugh to think of the good canon of Cezene, father Bungo, collecting into a quarto volume, entitled *De myſteriis numerorum*, all the ridiculous conceits which Nichomachus, Ptolemy, Porphyrius, and ſeveral more of the ancients, childiſhly publiſhed in regard to numbers. How could it enter the minds of reaſonable beings to aſcribe phyſical energy to things purely metaphyſical? For numbers are mere conceptions of the mind, and conſequently can have no influence in nature.

None, therefore, but old women, or perſons of weak minds, can believe in the virtues of numbers. Some entertain a notion, that if 13 perſons ſit at the ſame table, one of them will die in the courſe of the year; but there is more probability that one will die in the ſame time, if the number be 24.

The ſeries 1 2 3 4 5 6 7 9 is of ſuch a nature, that it may be multiplied by certain numbers, the product of which ſhall conſiſt of twos, threes, or fours, &c. at pleaſure.

To find a multiplier which ſhall give ſimilar figures in the product, 9 muſt be privately multiplied by 2, 3, or 4, according as it is required to have twos, threes, or fours, in the product.

For example, if it be required that the product ſhall contain only twos, we muſt multiply 9 privately by 2, which will give 18 for the multiplier of the ſeries. If we multiply the ſame number by 3, we ſhall have 27; if by 4, we ſhall have 36, &c.

If the ſeries, therefore, be multiplied ſucceſſively by 18, 27, 36, the products will be compoſed of twos, threes, fours, &c, as may be eaſily proved by trial.

If the number 37 be multiplied by any of the terms of the arithmetical progreſſion 3, 6, 9, 12, &c, all the products will conſiſt of ſimilar figures,

37	37	37
3	6	9
111	222	333

We may here obſerve, that the product 111, is compoſed of 37 multiplied by 3; or of 7 multiplied by 3, which makes 21, and 30 multiplied by 3 which makes 90; the ſum total being one and eleven tens, which, according to the laws of numbers, can be expreſſed only by three units.

It will not then appear aſtoniſhing that 37 multiplied by 6, ſhould give the double of theſe three units; and ſo of the reſt.

The number 5 has this peculiar property, that when multiplied by an odd number, the product will always terminate with 5; and if multiplied by an even number, will terminate with a cipher,

5	5	5	5
3	5	4	6
15	25	20	30

The number 9 has this property, that the ſum of the figures of every number, of which it is a multiple, forms a multiple of 9.

Thus, the product of 17 multiplied by 9 is 153, and the sum of theſe figures $1 + 5 + 3 = 9$. In the like manner, the ſum of the figures 6777, which is the product of 753 multiplied by 9, is equal to 27, a multiple of 9.

This will not appear aſtoniſhing when we reflect, that twice 9 is equal to 18, three times $9 = 27$, &c; where it is evident that the tens and units of the product are always reciprocal complements of 9.

If we take any two numbers whatever, one of them, or their ſum, or their difference, will always be diviſible by three. This may be ſo eaſily proved, that it is needleſs to illuſtrate it by examples.

Every ſquare number muſt neceſſarily terminate with two ciphers, or by 1, 4, 5, 6, 9; and this may enable us to determine, at one view, whether a numerical quantity be a ſquare or not. (10)

The number 2, of all the whole numbers, is the only one the ſum and product of which are equal. Thus $2 + 2 = 4$, as well as 2×2. But in fractional numbers we can find other two quantities, the ſum and product of which are in like manner equal.

For this purpoſe, the ſum of the two numbers muſt be divided by each of them ſeparately. The two fractions which thence ariſe, will give the ſame quantity, both when added and multiplied. (5, 7.)

If we take the two numbers 2 and 5, and divide their ſum by each of them ſeparately, the two fractions $\frac{7}{2}$, $\frac{7}{5}$, will give the ſame reſult when added, as well as when multiplied. This may be eaſily proved by any perſon in the leaſt acquainted with vulgar fractions. (2)

Every ſquare number (10) is diviſible by 3, or becomes ſo when diminiſhed by unity. This may be eaſily proved with any ſquare number whatever. Thus, $4 - 1$, $16 - 1$, $25 - 1$, $49 - 1$, $121 - 1$, &c, are all diviſible by 3; and the caſe is the ſame with the reſt.

Every ſquare number is diviſible by 4, or becomes ſo when diminiſhed by unity.

Every ſquare number is diviſible alſo by 5, or becomes ſo when increaſed or diminiſhed by unity.

Every odd ſquare, diminiſhed by unity, is a multiple of 8.

Every power of 5 terminates with 5, and every power of 6 with 6. (10)

PROB. *To find Two Numbers, the Squares of which, if added together, shall form a Square Number.*

If any two numbers whatever be multiplied together, the double of their product will be one of the two numbers sought; and the difference of their squares will be the other.

Thus, if the numbers 2 and 3, the squares of which are 4 and 9, be multiplied together, their product will be 6: if we then take 12, the double of this product, and 5 the difference of their squares, we shall have two numbers, the sum of the squares of which will be a square number; for their squares are 144 and 25, which by addition give 169, the square of 13.

Of Arithmetical and Geometrical Progression, with some Problems which depend on them.

§ 1. *Arithmetical Progression, with an Explanation of its Principal Properties.*

Any series of numbers, continually increasing or decreasing by the same quantity, forms what is called an arithmetical progression. (19)

Thus, the series of numbers 1, 2, 3, 4, 5, 6, &c, or 1, 5, 9, 13, &c, or 20, 18, 16, 14, 12, &c, or 15, 12, 9, 6, 3, are arithmetical progressions; for, in the first, the difference between each term and the following one, which exceeds it, is always

1; in the second it is 4; it is 2 also in the third, which goes on decreasing, and 3 in the fourth.

From this definition of arithmetical progression, the following consequences may be deduced.

1st. Any term of an arithmetical progression, is equal to the first, plus the common difference taken as many times as there are terms before it. (22)

2d. The sum of the extremes, is always equal to the sum of any two terms equally distant from them; or double the mean term, if the progression contains an odd number of terms. (21)

3rd. If the sum of the extremes be multiplied by half the number of terms, when the terms are even, or the mean by the whole number of terms when the latter are odd, the product will be the sum of the progression.

By considering with a little attention the following progressions, the truth of these consequences will be readily perceived:

$$\div\ 2 \cdot 5 \cdot 8 \cdot 11 \cdot 14 \cdot 17.$$

$$\div\ 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9.$$

PROB. I. *If a hundred stones are placed in a straight line, at the distance of a yard from each other, the first being at the same distance from a basket; how many yards must the person walk, who engages to pick them up, one by one, and to put them into the basket?*

It is evident that, to pick up the first stone, and put it into the basket, the person must walk two yards; for the second he must walk 4; for the third 6, and so on increasing by two, to the hundredth.

The number of yards, therefore, which the person must walk, will be equal to the sum of the progression 2, 4, 6, &c, the last term of which is 200 (22). But the sum of the progression is equal to 202, the sum of the two extremes, multiplied by 50, or half the number of terms; that is to say 10100 yards, which makes more than 5½ miles.

PROB. II. *A gentleman employed a bricklayer to sink a well, and agreed to give him at the rate of 3 shillings for the first yard in depth, 5 for the second, 7 for the third, and so on, increasing to the twentieth, where he expected to find water: how much was due to the bricklayer when he had completed the work?*

This question may be easily answered by the rules before given, for the difference of the terms is 2, and the number of terms 20; consequently, to find the twentieth term, we must multiply 2 by 19, and add 38, the product, to the first term 3, which will give for the twentieth term 41. (22)

If we then add the first and last terms, that is to say 3 and 41, which will make 44, and multiply this sum by 10, or half the number of terms, the product 440 will be the sum of all the terms of the progression, or the number of shillings due to the bricklayer, when he completed the work. (21)

He would therefore have to receive 22l.

PROB. III. *A merchant being considerably in debt, one of his creditors, to whom he owed* 1860*l*, *offered to give him an acquittance, on condition of his agreeing to pay the whole sum in twelve monthly installments; that is to say,* 100*l. the first month, and to increase the payment by a certain sum each succeeding month to the twelfth inclusive, when the whole debt would be discharged. By what sum was the payment of each month increased?*

In this problem, we have given the first term 100, the number of the terms 12, and their sum 1860; but the common difference of the terms is unknown.

This difference may be found in the following manner: As the sum of the extremes, in an even arithmetical progression, is equal to the sum total, divided by half the number of terms, if the sum total 1860 be divided by 6, or half the number of terms, we shall have 310 for the sum of the first and last term, from which if we subtract 100, the first term, the remainder 210 will be the last term; but the last term is always equal to the first and the common difference taken as many times as there are terms before it. If we, therefore, deduct the first term 100, from 210 the last, and divide 110, the remainder, by 11, we shall have 10 as the required difference. The first term being 100, the second therefore will be 110, the third 120, &c. (21)

PROB. IV. *A gentleman employed a bricklayer to sink a well, to the depth of 20 yards, and agreed to give him 20l. for the whole; but the bricklayer happening to die when he had completed only 8 yards, how much was due to his heirs?*

To imagine that two fiths of the whole sum were due to the workman, because 8 yards are two fifths of the whole depth, would be erroneous; for as the difficulty must increase arithmetically as the depth, it is natural to suppose that the price should increase in the same ratio.

To resolve this problem, therefore, 20l, or 400 shillings, must be divided into twenty terms in arithmetical progression; and the sum of the first eight of these will express what was due to the bricklayer for his labour.

But 400 shillings may be divided into twenty terms in arithmetical progression a great many different ways, according to the value of the first term, which is here undetermined: if we suppose it, for example, to be 1 shilling, the progression will be 1, 3, 5, 7, &c, the last term of which will be 39; and consequently the sum of the first eight terms will be 64 shillings.

But to resolve the problem in a proper manner, so as to give to the bricklayer his just due for the commencement of the work, we must determine what is the fair value of a yard of work similar to the first, and then assume that value as the first term of the progression. We shall here suppose that this value is 5 shillings; and in that case the required progression will be 5, $6\frac{11}{19}$, $8\frac{3}{19}$, $9\frac{14}{19}$, $11\frac{6}{19}$, $12\frac{17}{19}$, &c, the common difference of which is $1\frac{11}{19}$, and the last term is 35.

Now to find the eighth term, which is necessary before we can find the sum of the first eight terms, multiply the common difference $\frac{30}{19}$ by 7, which will give $11\frac{1}{19}$, and add this product to 5, the first term, which will give the eighth term $16\frac{1}{19}$; if we then add $16\frac{1}{19}$ to the first term, and multiply the sum, $21\frac{1}{19}$, by 4, the product, $84\frac{4}{19}$, will be the sum of the first eight terms, or what was due to the bricklayer for the part of the work he had compleated. The bricklayer, therefore, had to receive $84\frac{4}{19}$ shillings, or 4l. 4s. $2\frac{10}{19}$d.

§ 2. *Of Geometrical Progressions, and an Explanation of their principal Properties.*

If there be a series of numbers, each of which is the product of the preceeding by a common multiplier (18), these numbers form what is called a geometrical progression. Thus, 1, 2, 4, 8, 16, &c, form a geometrical progression; for the second is the double of the first, the third the double of the second, and so on in succession. The terms 1, 3, 9, 27, 81, &c, form also a geometrical progression, each being the triple of the preceding.

Progressions may be either increasing, as the two above mentioned, or decreasing as the two following 16, 8, 4, 2, 1; 81, 27, 9, 3, 1.

The principal property of geometrical progression is, that if we take any three following terms whatever, as 3, 9, 27, the product 81 of the extremes will be equal to the square of the mean 9. In like manner, if we take any four following terms, as 3, 9, 27, 81, the product 243, of the extremes, will be equal to that of the two means, 9 and 27.

In the last place, if any number of terms whatever of the series be assumed, as 2, 4, 8, 16, 32, 64, the product of the extremes, 2 and 64, will be equal to the product of any two terms equally distant from them, as 4 and 32, or 8 and 16. If the number of terms be odd, it is evident that there will be only one term equally distant from the two extremes; and in that case the square of that term will be equal to the product of the extremes, or of any two terms whatever equally distant from them, or from the mean. (23, 24)

Between geometrical and arithmetical progression, there is a certain analogy, which deserves here to be mentioned, and which is, that the same results are obtained in the former, by employing multiplication and division, as are obtained in the latter by addition and subtraction. When, in the latter, we take the half or the third, we employ in the former extraction of the square, cube, &c. roots.

Thus, to find an arithmetical mean between two numbers, for example 3 and 12, we must add the two extremes together, and take the half of 15 their sum, which is $7\frac{1}{2}$; but to find a geometrical mean between two numbers, we must multiply them together, and extract the square root of their product: for example, if a geometrical mean between 3 and 12 be required, we must extract the square root of their product 36, which will give 6; and, indeed, $\div\!\!\div$ 3 : 6 : 12. (23)

A geometrical progression may decrease *in infinitum*, without ever coming to 0; for it is evident that any part of a quantity whatever, greater than 0 can never become 0. A decreasing geometrical progression may be continued, therefore, *in infinitum*, since to find the following term, nothing will be necessary but to divide the last term by the exponent or common ratio. We shall here give two examples of decreasing geometrical progressions.

$\div\!\!\div$ $1 : \frac{1}{2} : \frac{1}{4} : \frac{1}{8} : \frac{1}{16} : \frac{1}{32} : \frac{1}{64}$, &c.

$\div\!\!\div$ $1 : \frac{1}{3} : \frac{1}{9} : \frac{1}{27} : \frac{1}{81} : \frac{1}{243}$, &c.

The sum of an increasing geometrical progression, continued *ad infinitum*, is evidently infinite; but that of a decreasing geometrical progression, whatever be the number of terms supposed, is always finite. Thus, the sum of the terms, continued *in infinitum*, of the geometrical progression $\div\!\!\div$ $1 : \frac{1}{2} : \frac{1}{4} : \frac{1}{8} : \frac{1}{16}$: &c, is only 2. That of the progression $\div\!\!\div$ $1 : \frac{1}{3} : \frac{1}{9} : \frac{1}{27} : \frac{1}{81}$: &c, is only $1\frac{1}{2}$.

Of harmonical Progreſſion.

Three numbers are in harmonical proportion, when the firſt is to the laſt as the difference between the firſt and the ſecond is to the difference between the ſecond and third. Thus the numbers 6, 3, 2 are in harmonical proportion; for 6 is to 2 as 3, the difference between the two firſt numbers, is to 1, the difference of the two laſt. This kind of relation is called harmonical, for a reaſon which will be ſeen hereafter.

When three numbers in decreaſing harmonical proportion are given, it is eaſy to find a fourth; nothing is neceſſary but to find a third harmonical to the two laſt, and this will be the fourth term required. In like manner, the third and fourth may be employed to find a fifth, and ſo on in ſucceſſion: this will form what is called a harmonical progreſſion, which by the above method may be always continued decreaſing.

If we ſuppoſe the two firſt numbers to be 2 and 1, we ſhall have the harmonical progreſſion 2, 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. It is a remarkable property, therefore, of the ſeries of fractions, having unity for their numerators, and for their denominators the numbers of the natural progreſſion, that they are in harmonical progreſſion.

This ſeries of numbers, indeed, contains all the muſical concords poſſible; for the ratio of 1 to $\frac{1}{2}$, gives the octave; that of $\frac{1}{2}$ to $\frac{1}{3}$, or of 3 to 2, the fifth; that of $\frac{1}{3}$ to $\frac{1}{4}$, or of 4 to 3, the fourth; that of $\frac{1}{4}$ to $\frac{1}{5}$, or of 5 to 4, the third major; that of $\frac{1}{5}$ to $\frac{1}{6}$, or of 6 to 5, the third minor; that of $\frac{1}{8}$ to $\frac{1}{9}$, or of 9 to 8, the tone major, and that of $\frac{1}{9}$ to $\frac{1}{10}$, or of 10 to 9, the tone minor. But this will be explained more at large when we come to treat of harmony.

Let us now return to geometrical progression, and the application of it to a few problems, which may serve as a rule for the solution of all others of the same kind.

PROB. I. *If Achilles can walk ten times as fast as a tortoise, which is a furlong before him, can crawl, will the former overtake the latter; and how far must he walk before he does so?*

This question has been thought worthy of notice merely because Zeno, the founder of the sect of the stoics, pretended to prove by a sophism, that Achilles would never overtake the tortoise; for while Achilles, said he, is walking a furlong the tortoise will have advanced the tenth of a furlong; and while the former is walking that tenth the tortoise will have advanced the hundredth part of a furlong, and so on *in infinitum*; consequently, an infinite number of instants must elapse before the hero can come up with the tortoise, and therefore he will never come up with it.

Any person, however, of common sense may readily perceive, that Achilles will soon come up with the tortoise. In what then consists the sophism? It may be explained as follows:

Achilles, indeed, would never overtake the tortoise, if the intervals of time, during which he is supposed to be walkng the first furlong, and then the tenth, hundredth, and thousandth parts of a furlong, which the tortoise has successively advanced before him, were equal; but if we suppose that he has walked the first furlong in 10 minutes, he will require only one minute to walk the tenth of a furlong, and $\frac{1}{10}$ of a minute to walk the hundredth, &c. The intervals of time, therefore, which Achilles will require to pass over the space gained by the tortoise during the preceding time, will go on decreasing in the following manner: 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c; and this series forms a sub-decuple geometrical progression, the sum of which is equal to $11\frac{1}{9}$, or the interval of time, at the end of which Achilles will have reached the tortoise.

PROB II. *If the hour and minute hands of a clock both begin to move exactly at noon, at what points of the dial-plate will they be successively in conjunction, during a whole revolution of the twelve hours?*

This problem, considered in a certain manner, is in nothing different from the preceding. The minute hand acts here the part which Achilles did in the former; and the hour hand, which moves twelve times ſlower, that of the tortoiſe. In the laſt place, if we ſuppoſe the hour hand to be beginning a ſecond revolution, and the minute hand to be beginning a firſt, the diſtance which the one has gained over the other will be a whole revolution of the dial-plate. When the minute hand has made one revolution, the hour hand will have made one twelfth of a revolution, and ſo on progreſſively. To reſolve the problem, therefore, we need only apply to theſe data the method employed in the former caſe, and we ſhall find, that the interval from noon to the point where the hands come again into conjunction will be $\frac{1}{11}$ of a whole revolution; or, what amounts to the ſame thing, one hour and $\frac{1}{11}$ of an hour. They will afterwards be in conjunction at 2 hours and $\frac{2}{11}$; 3 hours and $\frac{3}{11}$; 4 hours and $\frac{4}{11}$ &c; and, in the laſt place, at 11 hours and $\frac{11}{11}$, that is to ſay at 12 hours.

PROB. III. *A ſovereign being deſirous to confer a liberal reward on one of his courtiers, who had performed ſome very important ſervice, deſired him to aſk whatever he thought proper, aſſuring him it ſhould be granted. The courtier, who was well acquainted with the ſcience of numbers, only requeſted that the monarch would give him a quantity of wheat equal to that which would* *arise from one grain doubled sixty-three times successively. What was the value of the reward?*

It will be found by calculation, that the sixty-fourth term of the double progression ∺ 1 : 2 : 4 : 8 : 16 : 32 : &c, is 9223372036854775808. But the sum of all the terms of a double progression, beginning with unity, may be obtained by doubling the last term, and subtracting from it unity. The number of the grains of wheat, therefore, in the present case, will be 18446744073709551615. Now if a standard pint contains 9216 grains of wheat, a gallon will contain 73728 : and as eight gallons make one bushel, if we divide the above result by eight times 73728, we shall have 31274997411295, for the number of the bushels of wheat equal to the above number of grains, a quantity greater than what the whole surface of the earth could produce in several years, and which in value exceeds all the riches perhaps on the globe of the earth.

Another problem of the same kind is proposed in the following manner :

A gentleman taking a fancy to a horse, which a horse-dealer wished to dispose of at as high a price as he could, the latter, to induce the gentleman to become a puchaser, offered to let him have the horse for the value of the twenty-fourth *nail in his ſhoes, reckoning one farthing for the firſt nail, two for the ſecond, four for the third, and ſo on to the twenty-fourth. The gentleman, thinking he ſhould have a good bargain, accepted the offer: what was the price of the horſe?*

By calculating as before, the twenty-fourth term of the progreſſion ∻ 1 : 2 : 4 : 8 : &c. will be found to be 8388608, equal to the number of farthings the purchaſer gave for the horſe. The price, therefore, amounted to 8738*l.* 2*s.* 8*d.* which is more than any Arabian horſe, even of the nobleſt breed, was ever ſold for.

We ſhall conclude this article with ſome phyſico-mathematical obſervations on the prodigious fecundity and progreſſive multiplication of animals and vegetables, which would take place if the powers of nature were not continually meeting with obſtacles.

1ſt. It is not aſtoniſhing that the race of Abraham, after ſojourning 260 years in Egypt, ſhould have formed a nation capable of giving uneaſineſs to the ſovereigns of that country. We are told, in the ſacred writings, that Jacob ſettled in Egypt with ſeventy perſons: now, if we ſuppoſe, that among theſe ſeventy perſons, there were twenty too far advanced in life, or too young to have children; that of the remaining fifty, twenty-five were males and as many females, forming twenty-five married couples, and that each couple in the ſpace of twenty-five years, produced, one with another, eight children, which will not appear incredible in a country celebrated for the fertility of its inhabitants; we shall find that, at the end of twenty-five years, the above seventy persons may have increased to two hundred and seventy; from which if we deduct those who died, there will perhaps be no exaggeration in making them amount to two hundred and ten. The race of Jacob, therefore, after sojourning twenty-five years in Egypt, may have been tripled. In like manner, these two hundred and ten persons, after twenty-five years more, may have increased to six hundred and thirty; and so on in triple geometrical progression: hence it follows, that, at the end of two hundred and twenty-five years, the population may have amounted to 1377810 persons, among whom there might easily be five or six hundred thousand adults fit to bear arms.

2d. If we suppose that the race of Adam, making a proper deduction for those who died, may have been doubled every twenty years, which certainly is not inconsistent with the powers of nature, the number of men at the end of five centuries, may have amounted to 1048576. Now, as Adam lived about 900 years, he may have seen, therefore, when in the prime of life, a posterity of 1048576 persons.

3d. How great would be the multiplication of many animals, did not the difficulty of finding food, the continual war which they carry on against each other, or the numbers of them consumed by man, set bounds to their propagation? It might easily be proved, that the breed of a sow, which brings forth six young, two males and four females, if we suppose that each female produces every year after six young, four of them females and two males, would in twelve years amount to 33554230.

Several other animals, such as rabbits and cats, which go with young only for a few weeks, would multiply with still greater rapidity: in half a century the whole earth would not be sufficient to supply them with food, nor even to contain them.

If all the ova of a herring were fecundated, a very few years would be sufficient to make its posterity fill the whole ocean: for every oviparous fish contains thousands of ova, which it deposits in spawning time. Let us suppose that the number of ova amounts only to 2000, and that these produce as many fish, half males and half females; in the second year there would be more than 200000; in the third, more than 200000000; and in the eighth year, the number would exceed that expressed by 2 followed by twenty-four cyphers. As the earth contains scarcely so many cubic inches, the ocean, if it covered the whole globe, would not be sufficient to contain all these fish, the produce of one herring in eight years!

4th. Many vegetable productions, if all their seeds were put into the earth, would in a few years cover the whole surface of the globe. The hyosciamus, which of all the known plants produces, perhaps, the greatest number of seeds, would for this purpose require no more than four years. According to some experiments, it has been found that one stem of the hyosciamus produces sometimes more than 50000 seeds: now, if we admit the number to be only 10000, at the fourth crop it would amount to a 1 followed by sixteen cyphers. But as the whole surface of the earth contains no more than 5507634452576256 square feet, if we allow to each plant only one square foot, it will be seen that the whole surface of the earth would not be sufficient for the plants produced from one hyosciamus at the end of the fourth year!

Exercises in the Single and Compound Rule of Three, both Direct and Inverse.

(26) We shall confine ourselves to a small number of examples in this rule, which we briefly explained in the introduction.

Single Rule of Three Direct.

EXAMPLE I.

If 40 pioneers can dig a trench 268 yards long, in a certain time, how many yards can 60 pioneers dig in the same time?

$$40 : 60 :: 268 : x = 402. \quad (23)$$

EXAMPPE II.

If a ship with a fresh breeze, sails 200 leagues in three days, how long time will she require to sail 2000 leagues, every other circumstance being the same?

$$200 : 2000 :: 3 : x = 30.$$

EXAMPLE III.

If 52 yards 2 feet and 5 inches of mason work cost 168l. 9s. 4d. *what will be the expence of 77 yards 2 feet 8 inches of the like kind of work, at the same rate?*

To render the solution of this problem easier, the quantity of each piece of work must be reduced to inches, by multiplying the yards by 3 and 12; and, for the same reason, the price of the work must be reduced to pence. We shall then have the following proportion.

Inches.	Inches.	Pence.	
1901 :	2804 ::	40432 :	x. (23)

Single Rule of Three Inverse.

EXAMPLE I.

If 30 men can perform a certain piece of work in 25 days, how many men will be requisite to perform the same work in 10 days?

It is here evident that, as the work is to be done in a shorter time, it will require more men. Conſequently the proportion muſt be expreſſed in this manner:

Days. Days. Men.

$$10 : 25 :: 30 : x = 75.$$

EXAMPLE II.

A veſſel has proviſions for 15 days, but being obliged by certain circumſtances, to continue at ſea for 20 days, to what quantity muſt the daily ration of each man be reduced, to make the proviſions laſt during that time?

If the quantity of proviſions conſumed daily be repreſented by unity, it is evident that the reduced quantity muſt be as much below 1 as 15 days are leſs than 20. We ſhall therefore have

$$20 : 15 :: 1 : x = \tfrac{3}{4}.$$

Compound Rule of Three.

EXAMPLE I.

If 30 men perform 132 yards of work, in 18 days, how much will 54 men perform in 28 days?

Thirty men, working 18 days, will perform the ſame work as 18 times 30 men = 540, in one day; and, in like manner, 54 men, in 28 days, will perform the ſame work as 54 times 28 men = 1512, in one day. We have, therefore, the following proportion:

$$540 : 132 :: 1512 : x.$$

EXAMPLE II.

If a man, walking 7 hours a day, travels 230 leagues in 30 days; how many days would he require to perform a journey of 600 leagues, walking 10 hours a day?

This problem may be reduced to the single rule of three, if we consider, that to travel 30 days, employing 7 hours each day, is the same thing as to travel 30 times 7 hours, or 210 hours. The question therefore may be changed in the following manner: If 210 hours are required to travel 230 leagues, how many hours will be requisite to travel 600 leagues? When the number of hours which answer the question have been found, the required number of days may be found by dividing these hours by 10, as the traveller employs 10 hours each day. We must therefore find the fourth term of the proportion the first three of which are as follows:

Leagues.	Leagues.	Hours.	Hours.
230 :	600 ::	210 :	x.

Rule of Fellowship.

As this rule is merely an application of what has been said respecting the rule of three, we shall only give a few examples, to illustrate the use of it.

EXAMPLE I.

A privateer, belonging to three merchants, captured a prize, worth 80000l: what will each partner's ſhare amount to, the firſt having advanced to purchaſe and fit out the veſſel 2000l, the ſecond 6000l, and the third 12000l?

It is here evident, that each partner muſt have a ſhare of the prize proportioned to the money he advanced.

We muſt therefore make this proportion: As the ſum advanced by each partner, is to the whole money advanced, ſo is the ſhare of each to the whole prize. Hence we ſhall have the following proportions, where the ſecond and fourth terms, that is to ſay, the ſum total of the money advanced and the value of the prize, are common to all of them:

$$\left.\begin{matrix}2000\\6000\\12000\end{matrix}\right\} : 20000 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : 80000.$$

EXAMPLE II.

Three perſons having entered into partnerſhip, the firſt advanced 3000l. for 6 months; the ſecond 4000l. for 5 months, and the third 8000l. for 9 months; at the end of that time they found that their gain amounted to 150000l. how much will each partner's ſhare be worth?

As this problem belongs to the compound rule of three, we shall take, as the first terms, the product of the money advanced by each partner, multiplied by the time it was employed; and for the second, the sum of these products, in the following manner:

$$\left.\begin{matrix}18000\\20000\\72000\end{matrix}\right\} : 110000 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : 150000$$

It may be readily seen, that by means of the rule of three any sum, such as the amount of a legacy for example, may be easily divided among several persons, in such a manner, that the shares shall be in the ratio of certain determinate numbers, as 3, 4, 6. In this case, these numbers must be considered as three sums advanced by three partners, and their sum as the total of the money advanced: if we then call the legacy to be divided a, we shall have the following proportion:

$$\left.\begin{matrix}3\\4\\6\end{matrix}\right\} : 13 :: \left\{\begin{matrix}x\\x\\x\end{matrix}\right. : a.$$

Rule of Alligation.

Alligation is of two kinds. The first consists in finding the common price of several things, supposed to be mixed together; as, if a goldsmith, for example, should make a composition of gold, silver, &c. and be desirous to know the value of an ounce of this mixture. The ſame rule is employed to determine the mean price of ſeveral liquors, or different kinds of merchandiſe, mixed together.

This rule is exceedingly eaſy; for nothing is neceſſary, in ſolving queſtions of this kind, but to divide the whole value of the articles by the quantity of each article employed for the mixture, and the quotient will be the anſwer.

EXAMPLE I.

A wine merchant mixes together 200 bottles of Madeira, at 5 ſhillings; 500 of Port at 3s; 800 of Malaga at 4s; and 300 of Tokay at 8s: how much is a bottle of this mixture worth?

$$\begin{array}{rcl} 200 \times 5 & = & 1000 \\ 500 \times 3 & = & 1500 \\ 800 \times 4 & = & 3200 \\ 300 \times 8 & = & 2400 \\ \hline 1800 & & 8100\ \text{shil.} \end{array}$$

Now, if 8100s, or the whole value of the wine, be divided by 1800, the number of the bottles, the quotient will expreſs the value of each bottle of the mixture. Conſequently $\frac{8100}{1800} = \frac{81}{18} =$ 4s. 6d.

EXAMPLE II.

A gentleman employed 300 workmen, 50 of which were paid at the rate of 8s. a day; 70 at the rate *of 6s; and 180 at the rate of 4s: how much did each of them, taking one with another, cost him per day?*

The sum total, which is 1540s. must be divided by 300, the number of the workmen, and the quotient will be what each of them, taken one with another, cost him per day.

$$\frac{1540}{300} = \frac{154}{30} = 5s. \tfrac{2}{15}.$$

The object of the second kind of Alligation, is to determine in what proportion several things, of different values, ought to be mixed, in order to have an article of a certain mean price.

To obtain this result, the prices of the things to be mixed must be arranged, as seen in the following examples.

EXAMPLE I.

A grocer, who has tea at 3s. 4s. 7s. and 9s. per pound, is desirous of having a mixture which he can sell at 5s. per pound. In what proportions must he mix these four kinds of tea, so as to be able to sell the mixture at 5s?

Arrange the prices of the things to be mixed as seen at A; placing those which are greater than the mean price at the top, and those which are less at the bottom:

	3*s*.		2
	4*s*.		4
Fig. A.		5*s*.	
	7*s*.		2
	9*s*.		1
			—
			9

Then compare in succession with the mean prices the prices of all the things to be mixed, and place the differences as in the above example.

Thus, the difference between 3 and 5 is 2, which must be set down opposite to 7, and that between 4 and 5 is 1, which must be placed opposite to 9. Then proceed to the prices greater than that of the mean price, and compare them with that price in the following manner: The difference between 7 and 5 is 2, which place opposite to 3; and that between 9 and 5 is 4, which place opposite to 4.

The right hand column, the sum of which is 9, shews that to have 9 pounds of tea, at the mean price of 5s. the mixture must consist of 2 pounds at 3s. 4 lbs. at 4s. 2 lbs. at 7s. and 1 lb. at 9s.

It may here be readily seen, that 9 pounds of tea, at the mean price of 5s. will amount exactly to the value of the quantities mixed.

It may sometimes happen that the figures, expressing the different values of the things to be mixed, will not be equal in number both below and above the mean price; on this account, if there are, for example, three figures above it, and only two below, the difference of the third figure at the top must be placed opposite to the second at the bottom, along with the difference of the second at the top; and the difference of the second figure at the bottom must be set down twice: that is to say, it must be placed opposite to the second and the third at the top. See fig. B.

	2s.		1
	3s.		2
	4s.		2
Fig. B.		5s.	
	6s.		3
	7s.		2 + 1 = 3
			11

That is to say, to form 11 pounds of tea, of the mean price of 5s, one pound at 2s. two pounds at 3s. two pounds at 4s. three pounds at 6s. and three at 7s. must be mixed together.

EXAMPLE II.

A goldsmith has gold of 23 *carats fine, and some of* 13 *carats, which he is desirous of mixing, so as to form gold of* 18 *carats; what quantity of each must he take?* See Fig. C.

	13		5
Fig. C.		18	
	23		5
			10

It hence appears, that he must take an equal quantity of each.

By the same rule, we may find the quantity of alloy in any compound metal, for example bronze, which consists of copper and tin mixed together in a certain proportion. For this purpose, we must take three ingots of the same weight, one of bronze another of copper, and a third of pure tin. These three bodies, when weighed in water, will each lose a different part of their weight: the ingot of tin will lose more, and that of copper less, than the ingot of bronze. Let us suppose, that the loss of the bronze is 3 ounces, that of copper $2\frac{1}{2}$; and that of the tin $3\frac{1}{4}$. If these three numbers be arranged, according to the above formula, we shall have

$$\begin{array}{ccc} 2\frac{1}{2} & & \frac{1}{4} \\ & 3 & \\ 3\frac{1}{4} & & \frac{1}{2} \\ \hline & & \frac{3}{4} \end{array}$$

The sum of the two differences, $\frac{3}{4}$, shews that, in $\frac{3}{4}$ of bronze, there are one of copper and two of tin. This proportion being found, we may thence conclude that a mass of bronze similar to the ingot, weighing 150 pounds, would contain 100 pounds of tin, and 50 of copper.

A Chronological Problem.

How many years, months and days elapsed between the Battle of Marignan, fought on the 3d of

September 1515, and that of Fontenoi, fought on the 11th of May 1745?

The period from the Christian æra to the 3d of September 1515, comprehends 1514 years, 8 months and 3 days; and that from the same epoch to the 11th of May 1745, comprehends 1744 years, 4 months and 11 days.

Consequently, if we subtract the former from the latter, the difference, 229 years, 8 months and 8 days, will express the interval of time between the battle of Marignan and that of Fontenoi.

This method may be employed for every other problem of the like kind, and especially when it is necessary, in calculating interest, to know how many years, months and days have elapsed between certain dates.

The Rule of Tare.

By tare is commonly meant the weight of the cask, box or bag, in which goods are contained, and which being subtracted, when known, from the gross weight, leaves the real weight of the goods, called the net weight. In general, an allowance is made for it, at the rate of so much per hundred weight; and the quantity to be deducted is found by the Rule of Three, as in the following example:

A merchant purchases a bale of cotton, weighing 7 cwt. including the package, and is allowed at the rate of 16 per cwt. of tare: how much ought *to be deducted, on that account, from the gross weight of the bale of cotton?*

As the merchant purchases the goods by the net weight, the seller must give him 16 pounds over and above each cwt.; that is to say, for each 112 pounds, he must give him 128. We must, therefore, make the following proportion:

$$128 : 112 :: 700 : x. \quad (23)$$

The fourth term will express the number of pounds for which the merchant ought to pay.

Discount.

A merchant purchases goods to the ammount of 1000*l*, *to be paid at the end of a year; but the vender offers to abate* 10 *per cent. for ready money: how much must the buyer pay down?*

It might here be supposed, that the abatement ought to be as many times 10l. as 100 is contained in 1000; that is to say, that 100l. ought to be deducted, so that the merchant would have to pay only 900l.

But it is to be observed, that the vender ought to allow the purchaser only 10 per cent. on what he will really receive; that is to say, that every 110 pounds which the merchant has to pay, ought to be reduced to 100. We have, therefore, the following proportion:

$$110 : 100 :: 1000 : x.$$

This is the only true method of eſtimating diſcount; for if the vender received only 900l. ready money, this ſum, at 10 per cent. would produce, at the end of a year, no more than 990l. conſequently it would be much better for him to give a year's credit and receive 1000l.

Of Combinations and Permutations.

Before we enter on this ſubject, it will be neceſſary to explain the method of conſtructing a kind of table, treated of by Paſcal and others, called the arithmetical triangle; which is of great uſe to ſhorten calculations of this kind.

Firſt form a band A B of ten equal ſquares, and below it another C D of the like kind, but ſhorter by one ſquare on the left, ſo that it ſhall contain only nine ſquares; and continue in this manner, always making each ſucceſſive band a ſquare ſhorter. We ſhall thus have a ſeries of ſquares, diſpoſed in vertical and horizontal bands, and terminating at each extremity in a ſingle ſquare, ſo as to form a triangle, on which account it has been called the arithmetical triangle.

The numbers with which it is to be filled up, muſt be diſpoſed in the following manner:

In each of the ſquares of the firſt band inſcribe unity, as well as in each of thoſe in the diagonal A E.

A	1	1	1	1	1	1	1	1	1	1	B
	C	1	2	3	4	5	6	7	8	9	D
			1	3	6	10	15	21	28	36	
				1	4	10	20	35	56	84	
					1	5	15	35	70	126	
						1	6	21	56	126	
							1	7	28	84	
								1	8	36	
									1	9	
										1	
										E	

Then add the number in the firft fquare of the band C D, which is unity, to that in the fquare immediately above it, and write down the fum 2 in the following fquare. Add this number, in like manner, to that in the fquare above it, which will give 3, and write it down in the next fquare. By thefe means we fhall have the feries of the natural numbers 1, 2, 3, 4, 5, &c. The fame method muft be followed to fill up the other horizontal bands; that is to fay, each fquare ought always to contain the fum of the number in the preceding fquare of the fame row, and that which is immediately above it in the preceding. Thus, the number 15, which occupies the fifth ſquare of the third band, is equal to the ſum of 10, which ſtands in the preceding ſquare, and of 5, which is in the ſquare above it. The caſe is the ſame with 21, which is the ſum of 15 and 6; with 35, in the fourth band, which is the ſum of 15 and 20, &c.

The different ſeries of numbers, contained in this triangle, have different properties; but we ſhall here ſpeak only of thoſe which relate to combinations and permutations, as the reſt are of too abſtract a nature to be employed in arithmetical recreations, the principal object of which is to afford eaſy and agreeable amuſement.

There are two principal kinds of combination. The firſt is that where the different arrangements of ſeveral things are ſought, without any regard to their change of place.

The ſecond is that where regard is paid to the different changes of place. For example, the three quantities A, B, C, taken two and two, without regard to the different changes of place, are ſuſceptible of only three combinations A B, A C, B C; but if we pay regard to the changes of place, they are ſuſceptible of ſix combinations; for beſides the three former we ſhall have B A, C A, C B.

In combinations properly ſo called, no attention is paid to the order of the things. If four tickets, for example, marked A, B, C, D, were put into a hat, and any one should bet to draw out A and D, either by taking two at once, or one after the other, it would be of no importance whether A should be drawn first or last; the combinations A D and D A ought, therefore, to be here considered only as one.

But, if any one should bet to draw out A the first time, and D the second, the case would be very different, and it would then be necessary to attend to the order in which these four letters may be taken, and arranged together, two and two: it may be readily seen, that the different ways are A B, B A, A C, C A, A D, D A, B C, C B, B D, D B, C D, D C. In like manner, these four letters might be combined and arranged, three and three, twenty-four ways: as A B C, A C B, B A C, B C A, C A B, C B A, A D B, A B D, D B A, D A B, B A D, B D A, A C D, A D C, D A C, D C A, C A D, C D A, B C D, B D C, C B D, C D B, D B C, D C B. This is what is called permutation, or change of order.

PROB. I. *Any number of things whatever being given, to determine in how many different ways they may be combined, two and two, three and three, &c. without regard to order.*

This problem may be easily resolved by making use of the arithmetical triangle. Thus, for example, if there are eight things to be combined, three and three, we must take the ninth vertical band, or, in all cases, that band the order of which is expressed by a number exceeding by unity the number of the things to be combined; then the fourth horizontal band, or that the order of which is greater by unity than the number of the things to be taken together, and in the common square of both will be found the number of the combinations required; which, in the present example, will be 56.

But, as an arithmetical triangle may not always be at hand, or as the number of things to be combined may be too great to be found in such a table, the following simple method may be followed:

The number of the things to be combined, and the manner in which they are to be taken, viz. two and two, or three and three, being given:

1st. Form two arithmetical progressions, one in which the terms go on decreasing by unity, beginning with the given number of things to be combined, and the other consisting of the series of the natural numbers 1, 2, 3, 4, &c.

2d. Then take from each as many terms as there are things to be arranged together, in the proposed combination.

3d. Multiply together the terms of the first progression, and do the same with those of the second.

4th. In the last place, divide the first product by the second, and the quotient will be the number of the combinations required.

PROBLEM II. *In how many ways can 90 things be combined, two and two?*

According to the above rule, we must multiply 90 by 89, and divide the product, 8010, by the product of 1 and 2, that is 2; the quotient 4005 will be the number of combinations resulting from 90 things, taken two and two.

Should it be required to determine, in how many ways the same things can be combined three and three, the problem may be answered with equal ease; for we have only to multiply together 90, 89, 88, and to divide the product, 704880, by that of the three numbers 1, 2, 3; the quotient 117480 will be the number required.

In like manner, it will be found, that 90 things may be combined, four and four, 2555190 ways; for if the product of 90, 89, 88 and 87, be divided by 24, the product of 1, 2, 3, 4, we shall have the above result.

Were it asked how many conjunctions the seven planets could form with each other, two and two, we might reply 21; for, according to the general rule, if we multiply 7 by 6, which will give 42, and divide that number by the product of 1 and 2, that is 2, the quotient will be 21.

If we wished to know the number of all the conjunctions possible of these seven planets, two and two, three and three, &c.; by finding separately the number of the conjunctions two and two, then those three and three, &c. and adding them together, it will be seen that they amount to 120.

PROB. III. *Any number of things being given, to find in how many ways they can be arraged?*

This problem may be easily solved by following the method of induction; for,

1st. One thing can be arranged only in one way; in this case therefore the number of arrangements is = 1.

2d. Two things may be arranged together two ways; for with the letters A and B we can form the arrangements A B, and B A; the number of arrangements therefore is equal to 2, or the product of 1 and 2.

3d. The arrangements of three things, A, B, C, are in number six; for A B can form with C, the third, three different ones B A C, B C A, C B A, and there can be no more. Hence it is evident, that the required number is equal to the preceding multiplied by 3, or to the product of 1, 2, 3.

4th. If we add a fourth thing, for instance D, it is evident that, as each of the preceding arrangements may be combined with this fourth thing four ways, the above number 6, must be multiplied by 4, to obtain that of the arrangements resulting from four things; that is to say, the number will be 24, or the product of 1, 2, 3, 4.

It is needless to enlarge further on this subject; for it may be easily seen that, whatever be the number of the things given, the number of the arrangements they are susceptible of, may be found by multiplying together as many terms of the natural arithmetical progression as there are things proposed.

The following table will shew the immense number of permutations, or different arrangements, of which only 12 things are susceptible. We shall afterwards give the result of the permutations arising from the twenty-four letters of the alphabet.

Number of Things.	Permutations.
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600

Let it be required to assign what space would contain all the permutations of the twenty-four letters of the alphabet, supposing each of them to occupy a square line?

If we first suppose that each letter occupies a square line, a square inch will contain 144 letters; if we then multiply 144 by itself, the product 20736 will be the number of letters which can be contained in a ſquare foot; and if the laſt number be multiplied by 9, the product 186624 will be the number of letters which might be contained in a ſquare yard.

Now, as a mile is equal to 1760 yards, a ſquare mile will contain 3097600 ſquare yards; and if we multiply this number by 9, or the ſquare miles in a league, we ſhall have 27878400 for the number of ſquare yards in the ſquare league, and this number multiplied by 186624 will give 5202778521600 for the number of letters which could be contained in a ſquare league.

If this laſt product be multiplied by 21951022, or the ſquare leagues on the ſurface of the earth, allowing the diameter of it to be 7930 miles, we ſhall have 114206305788769075200 for the number of letters which, according to the above ſuppoſition, could be contained in this ſurface.

If we now employ the method already given to find the number of the permutations of the 24 letters, by ſucceſſively multiplying all the terms of the arithmetical progreſſion from 1 to 24, we ſhall have for the number of theſe permutations 620448401733239439360000, which is above five thouſand times greater than the number of letters that could be contained on the ſurface of the earth; and, as each permutation conſiſts of 24 letters, it thence follows, that to contain them, a ſpace 120000 times greater would be neceſſary. In attempting to form an idea of this immenſe ſurface, the imagination is, as it were, loſt; and it could hardly be believed that ſuch an extent would be required, were it not fully demonſtrated by calculation.

PROB. IV. *A club of ſeven perſons agreed to dine together every day ſucceſſively, as long as they could ſit down to table differently arranged. How many dinners would be neceſſary for that purpoſe?*

It may be eaſily found, by the rules already given, that the club muſt dine together 5040 times, before they would exhauſt all the arrangements poſſible, which would require above 13 years.

If any word be propoſed, ſuch as AMOR, and it be required to know how many different words could be formed of theſe four letters, which will give all the poſſible anagrams of that word, we ſhall find, by multiplying together 1, 2, 3 and 4, that they are in number 24, as repreſented in the following table.

AMOR	MORA	ORAM	RAMO
AMRO	MOAR	ORMA	RAOM
AOMR	MROA	OARM	RMAO
AORM	MRAO	OAMR	RMOA
ARMO	MAOR	OMRA	ROAM
AROM	MARO	OMAR	ROMA

The number of all the anagrams poſſible to be formed of one word, may be found in the ſame manner; but it muſt be confeſſed, that, if there were a great many letters in the word, the arrangements thence reſulting would be ſo numerous, as to require a long time to find them out.

Application of the doctrine of combinations to games of chance and to probabilities.

Though nothing, on the firſt view, ſeems more foreign to the province of the mathematics, than games of chance, the powers of analyſis have, as we may ſay, enchained this Proteus, and ſubjected it to calculation: it has found means to meaſure the different degrees of the probability of certain events, and this has given riſe to a new branch of mathematics, the principles of which we ſhall here explain.

When an event can take place in ſeveral different ways, the probabily of its happening in a certain determinate manner, is greater when, in the whole of the ways in which it can happen, the greater number of them determine it to happen in that manner. In a lottery, for example, every one knows that the probability of obtaining a prize is greater, according as the number of the prizes is greater, and as the whole number of the tickets is leſs. The probability of an event, therefore, is in the compound ratio of the number of cafes in which it can happen taken directly, and of the total number of thofe in which it can be varied, taken inverfely; confequently it may be expreffed by a fraction, having for its numerator the number of the favourable chances, and for its denominator the whole of the chances.

Thus, in a lottery containing 1000 tickets, 25 of which only are prizes, the probability of obtaining a prize will be reprefented by $\frac{25}{1000}$, or $\frac{1}{40}$; if there were 50 prizes, the probability would be double; for in that cafe it would be equal to $\frac{1}{20}$; but if the number of tickets, inftead of 1000, were 2000, the probability would be only one half of the former, or $\frac{1}{80}$; and if the whole number of the tickets were infinitely great, the prizes remaining the fame, it would be infinitely fmall, or 0; while, on the other hand, it would become certainty, and be expreffed by unity, if the number of the prizes were equal to that of the tickets.

Another principle of this theory, the truth of which may be readily perceived, and which it is neceffary here to explain, is as follows:

A perfon plays an equal game when the money ftaked, or rifked, is in the direct ratio of the probability of winning; for to play an equal game, is nothing elfe than to depofit a fum fo proportioned to the probability of winning, that, after a great many throws, the player may find himfelf nearly at par;

but for this purpose, the stakes must be proportioned to the probability which each of the players has in his favour. Let us suppose, for example, that A bets against B on a throw of the dice, and that there are two chances in favour of the former, and one for the latter: the game will be equal, if, after a great number of throws they separate nearly without any loss. But as there are two chances in favour of A, and only one for B, after 300 throws A will have won nearly 200, and B 100. A, therefore, ought to deposit 2 and B only 1; for by these means A, in winning 200 throws, will get 200; and B, in winning 100, will get 200 also. In such cases therefore it is said, that there is two to one in favour of A.

PROB. I. *In tossing up, what probability is there of throwing a head several times successively, or a tail; or, in playing with several pieces, what probability is there that they will all come up heads at one throw?*

As this game is well known, it is needless here to give any explanation of it; we shall therefore proceed to analyze the problem.

1st. It is evident that, as there is no reason why a head should come up rather than a tail, or a tail than a head, the probability of one of them coming up is equal to $\frac{1}{2}$, or an equal bet may be taken on either side.

But, if any one ſhould bet to bring heads ſucceſſively in two throws, to know what in this caſe ought to be ſtaked on each ſide, we muſt obſerve, that all the combinations poſſible of head and tail, which can take place, in two ſucceſſive throws with the ſame piece, are *head, head; head, tail; tail, head; tail, tail;* one of which only gives *head, head.* Here then there is only 1 caſe in 4 favourable to the perſon who bets to throw a head twice in ſucceſſion; the probability therefore of this event will be only $\frac{1}{4}$; and he who bets that it will take place, ought to depoſit only a crown, while his antagoniſt depoſits three: for the latter has three chances of winning, whereas the former has only one. To play an equal game, the money depoſited by each ought to be in this proportion.

It will be found, in like manner, that he who ſhould bet to bring a *head*, for example, three times ſucceſſively, would have in his favour only one of the eight combinations of *head* and *tail*, which might reſult from three ſucceſſive throws of the ſame piece. The probability therefore of this event would be $\frac{1}{8}$; while that of his adverſary would be $\frac{7}{8}$, and conſequently to play an equal game, he ought to bet only 1 to 7.

It is needleſs to go over all the other caſes; for it may be readily ſeen that the probability of throwing a head four times ſucceſſively would be only $\frac{1}{16}$, and ſo on. We ſhall ſay nothing farther therefore on the different combinations which might result from head and tail; as in all such cases the following general rule may be employed.

When the probability of two or more individual events are known, the probability of their taking place all together may be found, by multiplying together the probabilities of those events considered individually.

Thus, the probability of throwing a head, considered individually, being expressed at each throw by $\frac{1}{2}$, that of throwing it twice successively, will be $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; that of throwing it three times successively, will be $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$, and so on.

2d. The probability of throwing all *heads* or all *tails* with two, three, or four pieces, may be determined in the same manner. When two pieces are employed, there are 4 combinations of head and tail, only one of which is both *heads;* when three pieces are tossed up, at the same time, there are 8, one of which only is all *heads;* and so on. The probability, therefore, in each of these cases, is similar to those already examined.

It may be seen, indeed, without the help of analysis, that these two questions are absolutely the same, as may be proved in the following manner. To toss up the two pieces A and B at the same time, or to throw up the one after the other, when A the first has had time to settle, is certainly the same thing. Let us suppose then, that when A the first has settled, instead of tossing up B, the second, A, is taken from the ground, in order to be tossed up a second time: this will certainly be the same thing as if the piece B had been employed; for by the supposition they are both equal and similar, at least in regard to the chance of a *head* or a *tail* coming uppermost; consequently to toss up the two pieces A, B, at once, or to toss up twice successively the piece A, is the same thing.

3d. If it were asked, how much a person might bet to bring a *head* at least once in two throws, it may be found by the above method that the chance is 3 to 1. In two throws there are four combinations, three of which give at least one *head*, while there is only one which gives two *tails*; and hence it follows that there are three combinations in favour of the person who bets to bring a *head* once in two throws, and only one against him.

PROB. II. *Any number of dice being given, to determine the probability of throwing with them an assigned number of points.*

We here suppose that the dice are of the usual kind, that is to say, having six faces marked with the numbers 1, 2, 3, 4, 5, 6. This being premised, we shall analyze some of the first cases of the problem, that we may proceed gradually to those which are more complex.

1st. It is proposed to throw a determinate point, for example 6, with one die.

As the die has six faces, one of which only is marked six, and as any one of these may come up as readily as another, it is evident that there are 5 chances against the person who undertakes to throw 6 at one throw, and only 1 in his favour. To play an equal game he ought, therefore, to bet no more than 1 to 5.

2d. Let it be proposed to throw the same point 6, with two dice.

To analyze this case, it must first be observed that two dice give 36 different combinations; for each of the faces of the die A, for example, may combine with each of those of B, which will produce 36 combinations. We must next examine in how many ways the point 6 can be thrown with two dice. 1st. It will be found that it can be thrown by 3 and 3; 2d. By throwing 2 with the die A, and 4 with the die B, or 4 with A and 2 with B, which, as may be readily seen, forms two distinct cases; 3d. By throwing 1 with A and 5 with B, or 1 with B and 5 with A, which likewise forms two cases: these are evidently all the ways that can be found. Consequently, there are 5 cases favourable in the 36; and, therefore, the probability of throwing 6, with two dice, is $\frac{5}{36}$; and that of not throwing it $\frac{31}{36}$; hence it appears that the money ſtaked by the players ought to be in the ratio of theſe two fractions.

By analyſing the other caſes it will be found, that of throwing 2 with two dice, there is 1 chance in 36; of throwing 3, there are 2; of throwing 4, there are 3; of throwing 5, there are 4; of throwing 6, there are 5; of throwing 7, there are 6; of throwing 8, there are 5; of throwing 9, there are 4; of throwing 10, there are 3; of throwing 11, there are 2; and of throwing 12, or ſixes, there is 1.

If three dice were propoſed, with which the leaſt point that could be thrown is evidently 3, and the greateſt 18, it will be found, by a ſimilar analyſis, that in 216 different throws poſſible, with three dice, there is 1 chance of throwing 3, 3 of throwing 4, 6 of throwing 5, and ſo on; as may be ſeen in the annexed table, the uſe of which is as follows.

If it be required, for example, in how many different ways 13 can be thrown with three dice; look in the firſt vertical column, on the left, for the number 13, and at the top of the table for that indicating the number of the dice, and in the common ſquare of both, oppoſite to 13, will be found 21, or the number of ways in which 13 can be thrown with 3 dice. It will be found, in like manner, that with 4 dice, it may be thrown 140 ways; with 5 dice, 420 ways; and ſo on.

Table of the different ways in which any point can be thrown with one, two, three, or more dice.

NUMBER OF POINTS.	NUMBER OF THE DICE. I.	II.	III.	IV.	V.	VI.
1	1					
2	1	1				
3	1	2	1			
4	1	3	3	1		
5	1	4	6	4	1	
6		5	10	10	5	1
7		6	15	20	15	6
8		5	21	35	35	21
9		4	25	56	70	56
10		3	27	80	126	126
11		2	27	104	205	252
12		1	25	125	305	456
13			21	140	420	756
14			15	146	540	1161
15			10	140	651	1666
16			6	125	735	2247
17			3	104	780	2856
18			1	80	780	3431
19				56	735	3906
20				35	651	4221
21				20	540	4332
22				10	420	4221
23				4	305	3906
24				1	205	3431
25					126	2856

When once it is known in how many ways any point can be thrown with a certain number of dice, it will be easy to determine the probability of throwing it. Nothing will be necessary but to form a fraction, having for its numerator the number of ways in which the point can be thrown, and for its denominator the number 6, raised to that power denoted by the number of dice; for example, the cube of 6, or 216, for 3 dice; the biquadrate, or 1296, for 4; and so on.

Thus, the probability of throwing 13 with 3 dice is $\frac{21}{216}$; that of throwing it with 4 is $\frac{140}{1296}$.

Various other questions of the like kind might be proposed, some of which we shall here analyze.

PROB. III. *When two persons are playing, to determine the advantage or disadvantage on the side of the one who undertakes to throw a certain face, for example, that marked 6, in a certain number of throws.*

Let us first suppose that the person undertakes it at one throw. To determine the probability of his succeeding, we must consider, that he who holds the die has only one chance of winning, and that there are five of his losing: consequently to undertake it at one throw, he ought to bet only 1 to 5. There is, therefore, great disadvantage in undertaking on an even bet to throw 6 at one throw.

To determine the probability of throwing at least one face marked 6, in two throws, with the same die; we must observe, as has been already said in regard to tossing up, that this is the same thing as to undertake to bring one face marked 6, by throwing two dice at the same time. In this case, he who holds the dice has only 11 chances, or combinations, in his favour; for he may bring 6 with the first die, and 1, 2, 3, 4, or 5, with the second; or 6 with the second die, and 1, 2, 3, 4, or 5, with the first; or 6 with each die. But there are 25 chances, or combinations, against his winning, as may be seen in the following table:

1, 1	2, 1	3, 1	4, 1	5, 1
1, 2	2, 2	3, 2	4, 2	5, 2
1, 3	2, 3	3, 3	4, 3	5, 3
1, 4	2, 4	3, 4	4, 4	5, 4
1, 5	2, 5	3, 5	4, 5	5, 5

Hence it may readily be concluded, that he who undertakes to throw a 6 with two dice, ought to bet only 11 to 25; and consequently, that there is a disadvantage in undertaking it on an even bet.

We must here observe that 36, or the whole of the chances possible with one throw of two dice, is the square of the given number 6, the number of the faces of one die; and that 25, the number of the chances unfavourable to the person who undertakes to throw a determinate face, is the square of the number 6 diminiſhed by unity, that is of 5; for this reaſon, the number of the favourable chances in the preſent caſe, is the difference of the ſquares of 36 and 25, or of the ſquare of the number of the faces of the die, and of that of the faces of the ſame die leſs 1.

To determine the probability of throwing a 6 in three throws of the ſame die; we muſt, in like manner, conſider, that this is the ſame thing as to undertake to bring at leaſt one 6 by throwing three dice; but of the 216 different combinations produced by 3 dice, there are 125 in which there is no 6, and 91 where there is at leaſt one 6: conſequently, he who bets on throwing one 6, either in three throws with one die, or in one throw with three dice, ought to bet no more than 91 to 125; and it would be diſadvantageous to undertake it on an equal bet.

We muſt again obſerve, that 91 is the difference of the cube of the number of the faces of one die, viz. 216, and of 125 the cube of the ſame number diminiſhed by unity, that is to ſay of 5. Hence it may be ſeen, that to determine, in general, the probability of throwing any aſſigned face in a certain number of throws, or in one throw with a certain number of dice, we muſt raiſe 6, the number of the faces of one die, to that power indicated by the number of throws given, or of the dice to be thrown at once, and that we muſt then raiſe to the ſame power 6 leſs unity, that is to ſay 5, and ſubtract it from the former: the remainder, and this power of 5, will be the reſpective number of chances for winning or loſing.

For example, if a perſon ſhould bet to bring at leaſt one 3 with four dice; we muſt raiſe 6 to the biquadrate or fourth power, which is 1296, and ſubtract from it 625, which is the fourth power of 5: the remainder 671 will be the number of chances favourable for winning; and 625 will be thoſe of loſing: conſequently, there will be an advantage in laying an even bet.

There will be ſtill more advantage in undertaking, on an even bet, to throw a determinate point, for example 3, in five throws, or with five dice; for if we deduct the fifth power of 5, which is 3125, from 7776, the fifth power of 6, the remainder 4651 will be the number of the favourable chances, and 3125 that of the unfavourable.

Conſequently, to play an equal game, he who bets ought to depoſit 4652 to 3125, or about 3 to 2.

PROB. IV. *In how many throws may a perſon, with an equal chance of winning, bet to bring a determinate doublet, for example ſixes, with two dice?*

We already know that the probability of not throwing fixes with two dice is $\frac{35}{36}$; confequently, the probability of not throwing them, in two throws, will be as the fquare of that fraction; in three throws, as the cube; and fo on; but as the powers of any number ever fo little greater than unity go on always increafing, thofe of a number ever fo little lefs go on always decreafing; confequently the confecutive powers of $\frac{35}{36}$ will go on always decreafing. Let us conceive $\frac{35}{36}$ raifed to fuch a power, that it fhall be equal to $\frac{1}{2}$; now it will be found that the twenty-fourth power of $\frac{35}{36}$ is a little greater than $\frac{1}{2}$; and that the twenty-fifth power of the fame fraction is a little lefs than $\frac{1}{2}$; a perfon may then lay an even bet, with fome advantage, that another will not bring fixes in 24 throws, but an even bet cannot be laid with advantage that fixes will not come up in 25 throws. Confequently there is a difadvantage in laying an even bet to bring fixes in 24 throws; and, on the other hand, he who lays an even bet to throw fixes in 25 throws, does fo with advantage.

PROB. V. *What probability is there of throwing a determinate doublet, for example two threes, at one throw, with two or more dice?*

To determine the probability in this cafe, we muft confider, that in undertaking to throw two threes with 2 dice, there is only 1 favourable chance in the 36 given by 2 dice. Whence it follows, that the perſon who undertakes it, ought to bet only 1 to 35; if 3 dice were propoſed, we ſhould find that the bet ought to be 16 to 216; for the number of chances or combinations poſſible with 3 dice is 216; but when it is required to throw two threes with 3 dice, it may be done 16 different ways; for of the 36 combinations of the dice A and B, all thoſe in which there is only one 3, as 1, 3; 3, 1; &c. which are 10 in number, by combining with the face marked 3 of the die C, will conſequently give two threes. Beſides, the combination 3, 3 of the dice A, B, by combining with one of the ſix faces of the third C, will alſo give two threes; and hence there are 16 different ways of throwing two threes with 3 dice, which gives 16 favourable chances in 216. Conſequently, the probability of throwing two threes with 3 dice, is $\frac{16}{216}$; and therefore the perſon who undertakes it, ought to bet no more than 16 to 216, or nearly 2 to 27.

If the probability of throwing two threes with 4 dice be required, we ſhall find that it is expreſſed by $\frac{171}{1296}$; for, of the 1296 combinations ariſing from the faces of 4 dice, there are 150 which give 2 threes, 20 which give 3 threes, and 1 which gives 4, making altogether 171 throws, in which there are either 2, 3, or 4 threes. Conſequently, a perſon ought to bet no more than 19 to 144, or about 1 to 7$\frac{1}{2}$, on throwing at leaſt 2 threes with 4 dice.

In the laſt place, if the probability of throwing any doublet with ten dice, or more, at one throw, be required; it will be eaſy to determine it by the ſame method of calculation. For in the caſe of an indeterminate doublet, it is evident that the probability is ſix times as great as when a particular doublet is aſſigned; and therefore nothing will be neceſſary but to multiply the above probabilities by 6. The probability therefore with 2 dice, is $\frac{6}{36}$ or $\frac{1}{6}$; with 3 dice, $\frac{96}{216} = \frac{4}{9}$; with 4 dice $\frac{576}{216} = \frac{8}{3}$; ſo that there is an advantage in laying an even bet, to bring at leaſt one doublet with 4 dice.

PROB. VI. *Two perſons depoſit a certain ſum of money, and agree, that he who firſt gets a certain number of games, for example 3, ſhall have the whole; one of them has got two games, and the other one; but being unwilling to continue their play, they reſolve to divide the ſtake, in what manner muſt this be done?*

This problem is one of the firſt which engaged the attention of Paſcal, when he began to ſtudy the calculation of probabilities. He propoſed it to M. de Fermat, a celebrated geometrician of that period, who reſolved it by a different method, viz. that of combinations. We ſhall here give both.

It is evident that each of the players, when he depoſited his money, reſigned all right to it; but, on the other hand, each had a right to that which chance might give him; consequently, when they give over playing, the stake ought to be divided according to the probability each had of winning.

Case 1.

This proportion may be determined by the following mode of reasoning. Since the first player wants one game to be out, and the second two, it may be readily perceived that if they continued their play, and if the second won one game, he would want, in the same manner as the first, one game to be out; and if both players were equally advanced, their hopes of gaining the whole would be equal: in this supposition, therefore, they would have an equal right to the stake, and consequently each ought to have an equal share of it. It is evident therefore, that if the first wins the game about to be played, the whole stake will belong to him; and if he loses it he will be entitled only to one half. As the one case is as probable as the other, the first has a right to the half of these sums taken together; but together they make $\frac{3}{2}$, the half of which is $\frac{3}{4}$. Such is the portion of the stake belonging to the first player, and consequently that belonging to the second is only $\frac{1}{4}$.

Case 2.

The solution of the first case will enable us to resolve the second, in which we suppose that the first player wants one game to be out, and the second three; for if the firſt ſhould win one game, the whole of the ſtake would belong to him, and if he ſhould loſe one, ſo that the ſecond ſhould want only two games to be out, $\frac{3}{4}$ of the money would belong to the former, ſince they would then be in the ſituation alluded to in the firſt caſe. But as both theſe events are equally probable, the firſt ought to have the half of theſe two ſums taken together, or the half of $\frac{7}{4}$, that is to ſay $\frac{7}{8}$: the remainder, $\frac{1}{8}$, will be what ought to belong to the ſecond.

Caſe 3.

It will be found, by reaſoning in the ſame manner; if we ſuppoſe two games wanting to the firſt player, and three to the ſecond, that on ceaſing to play they ought to divide the ſtake in ſuch a manner, that the firſt may have $\frac{11}{16}$, and the ſecond $\frac{5}{16}$.

Caſe 4.

If they had agreed to play four games, and if the firſt wanted only two games, while the ſecond wanted four, the ſtake ought to be divided in ſuch a manner, that the firſt might have $\frac{13}{16}$, and the ſecond $\frac{3}{16}$.

We ſhall now explain the ſecond method of reſolving queſtions of this kind, which is that of combinations.

To reſolve, for example, the fourth caſe, in which we ſuppoſe that the firſt player wants two games to be out, and the ſecond four, ſo that both together want ſix games; if we ſubtract unity from that ſum, we ſhall have 5, which indicates, that we muſt take theſe five ſimilar letters *aaaaa*, favourable to the firſt player, and the five followigg, *bbbbb*, favourable to the ſecond. Theſe muſt be combined together as ſeen in the following table, where, of 32 combinations, the firſt 26, towards the left, where *a* occurs at leaſt twice, indicate the number of chances favourable to the firſt, and the 6 laſt, towards the right, where *a* is found at moſt only once, indicate the number of chances favourable to the ſecond.

a a a a a	*a a a b b*	*a a b b b*	*a b b b b*
a a a a b	*a a b b a*	*a b b b a*	*b b b b a*
a a a b a	*a b b a a*	*b b b a a*	*b a b b b*
a a b a a	*b b a a a*	*a b a b b*	*b b a b b*
a b a a a	*a a b a b*	*a b b a b*	*b b b a b*
b a a a a	*a b a a b*	*b b a a b*	*b b b b b*
	b a a a b	*b a a b b*	
	b a a b a	*b a b b a*	
	b a b a a	*b b a b a*	
	a b a b a	*b a b a b*	

Thus the hope of the firſt player will be to that of the ſecond, as 26 to 6, or as 13 to 3.

To reſolve the caſe in which we ſuppoſe that one of the players has won three games, and the other none; as he will be the winner who ſooneſt gets, four games, he muſt take unity from 5, the number of games wanting to both, which will give 4, and then examine in how many ways the letters *a* and *b* can be combined, four and four. These ways are in number 16, viz:

a a a a	*a a b b*	*a b b b*	*b b b b*
a a a b	*a b a b*	*b a b b*	
a a b a	*b a a b*	*b b a b*	
a b a a	*a b b a*	*b b b a*	
b a a a	*b a b a*		
	b b a a		

But, of these 16 combinations, it is evident there are 15 in which *a* is found at least once; and hence it appears that there are 15 combinations or chances favourable to the first player, and only one to the second; consequently they ought to share the stake in the ratio of 15 to 1; or the first ought to have $\frac{15}{16}$, and the second $\frac{1}{16}$.

PROB. VII. *A mountebank, at a country fair, amused the populace with the following game: he had 6 dice, each of which was marked only on one face, the first with 1, the second with 2, and so on to the sixth, which was marked 6; the person who played, gave him a certain sum of money, and he engaged to return it a hundred fold if, in throwing these six dice, the six marked faces should come up only once in 20 throws?*

Though the proposal of the mountebank does not, on the first view, appear very disadvantageous to thoſe who entruſted him with their money, it is certain that there were a great many chances againſt them.

It may indeed be ſeen that, of the 46656 combinations of the faces of 6 dice, there is only one which gives the 6 marked faces uppermoſt; the probability therefore of throwing them, at one throw, is expreſſed by $\frac{1}{46656}$: and as the adventurer was allowed 20 throws, the probability of his ſucceeding was only $\frac{20}{46656}$, which is nearly equal to $\frac{1}{2332}$. To play an equal game therefore, the mountebank ſhould have engaged to return 2332 times the money depoſited.

PROB. VIII. *The ſame mountebank offered a new chance to the perſon who had loſt, on the following conditions: to depoſit a ſum equal to the former, and to receive both the ſtakes in caſe he ſhould bring all the blank faces, in 3 ſucceſſive throws.*

Thoſe unacquainted with the method to be purſued in order to reſolve ſuch problems, are liable to reaſon in an erroneous manner reſpecting dice of this kind; for obſerving that there are five times as many blank as marked faces, they thence conclude that it is 5 to 1 that the perſon who throws them will not bring any point. They are, however, miſtaken, as the probability, on the contrary, is 2 to 1 that they will not come up all blank.

If we take only one die, it is 5 to 1 that the perſon who holds it will throw a blank; but if we add a ſecond die, it may be readily ſeen, that the marked face of the firſt may combine with each of the blank faces of the ſecond, and the marked face of the ſecond with each of the blank faces of the firſt; and, in the laſt place, the marked face of the one with the marked face of the other: conſequently, of the 36 combinations of the faces of theſe two dice, there are 11, in which there is at leaſt one marked face. But, as we have already obſerved, this number 11 is the difference of the ſquare of 6, the number of the faces of one die, and of the ſquare of the ſame number diminiſhed by unity, that is to ſay of 5.

If a third die be added, we ſhall find, by the like analyſis, that, of the 216 combinations of three dice, there are 91 in which there is at leaſt one marked face; and 91 is the difference of the cube of 6 or 216, and the cube of 5 or 125; the reſult will be the ſame in regard to the more complex caſes; and hence we may conclude that, of the 46656 combinations of the faces of the 6 dice in queſtion, there will be 31031 in which there is at leaſt one marked face, and 15625 in which all the faces are blank; conſequently the chance is 2 to 1 that ſome point, at leaſt, will be thrown; whereas, by the above reaſoning, it would appear that 5 to 1 might be betted on the contrary being the caſe.

Prob. ix. *In how many throws, with ſix dice, marked on all their faces, may a perſon engage, for an even bet, to throw* 1, 2, 3, 4, 5, 6?

We have juſt ſeen that there are 46655 chances to 1 that a perſon will not throw theſe 6 points with dice marked only on one of their faces; but the caſe is very different with 6 dice marked on all their faces; and to prove it, we need only to obſerve that the point 1, for example, may be thrown by each of the dice, as well as the 2, 3, &c. which renders the probability of theſe ſix points, 1, 2, 3, &c. coming up, much greater.

But to analize the problem more accurately, we ſhall obſerve that there are 2 ways of throwing 1, 2, with 2 dice; viz. 1 with the die A, and 2 with the die B; or 1 with the die B, and 2 with A. If it were propoſed to throw 1, 2, 3, with 3 dice; of the whole of the combinations of the faces of 3 dice, there are 6 which give the points 1, 2, 3; for 1 may be thrown with the die A, 2 with B, and 3 with C; or 1 with A, 2 with C, and 3 with B; or 1 with B, 2 with A, and 3 with C; or 1 with B, 2 with C, and 3 with A; or 1 with C, 2 with A, and 3 with B; or 1 with C, 2 with B, and 3 with A.

It hence appears, that to find the number of ways in which 1, 2, 3 can be thrown with 3 dice, 1, 2, 3

[illegible]

must be multiplied together. In like manner, to find the number of ways in which 1, 2, 3, 4 can be thrown with 4 dice, we must multiply together 1, 2, 3, 4, which will give 24; and, in the last place, to find in how many ways 1, 2, 3, 4, 5, 6 can be thrown with 6 dice, we must multiply together these six numbers, the product of which will be 720.

If the number 46656, which is the combinations of the faces of 6 dice, be divided by 720, we shall have $64\frac{4}{5}$ for the chances to 1, that these points will not come up at one throw; and consequently a person may undertake for an even bet to bring them in 64 throws.

In the last place, as the dice may be thrown 130 times, and more, in a quarter of an hour, a person may, with advantage, bet more than 2 to 1, that they will come up in the course of that time.

He who engages for an even bet to throw these points, in a quarter of an hour, undertakes what is highly advantageous to himself, and equally disadvantageous to his adversary.

Arithmetical Amusements in Divination and Combination.

PROB. 1. *To tell the number thought of by a person.*

Desire the person, who has thought of a number, to triple it, and to take the exact half of that triple if it be even, or the greater half if it be odd. Then desire him to triple that half, and ask him how many times it contains 9; for the number thought, if even, will contain twice as many units as it does nines, and one more if it be odd.

Thus, if 5 has been the number thought of, its triple will be 15, which cannot be divided by 2 without a remainder. The greater half of 15 is 8; and if this half be multiplied by 3, we ſhall have 24, which contains 9 twice: the number thought of will therefore be 4 plus 1, that is to ſay 5.

II.

Bid the perſon multiply the number thought of by itſelf; then deſire him to add unity to the number thought of, and to multiply it alſo by itſelf; in the laſt place aſk him to tell the difference of theſe two products, which will certainly be an odd number, and the leaſt half of it will be the number required.

Let the number thought of, for example, be 10, which multiplied by itſelf gives 100; in the next place, 10 increaſed by 1 is 11, which multiplied by itſelf makes 121; and the difference of theſe two ſquares is 21, the leaſt half of which being 10, is the number thought of.

This operation might be varied by deſiring the perſon to multiply the ſecond number by itſelf, after it has been diminiſhed by unity. In this caſe, the number thought of will be equal to the greater half of the difference of the two ſquares.

Thus, in the preceding example, the ſquare of the number thought of is 100, and that of the ſame number leſs unity is 81: the difference of theſe is 19, the greater half of which, or 10, is the number thought of.

III.

Bid the perſon take 1 from the number thought of, and then double the remainder; deſire him to take 1 from this double, and to add to it the number thought of: in the laſt place, aſk him the number ariſing from this addition, and if you add 3 to it, the third of the ſum will be the number thought of.

The application of this rule is ſo eaſy that it is needleſs to illuſtrate it by an example.

IV.

Deſire the perſon to add 1 to the triple of the number thought of, and to multiply the ſum by 3; then bid him add to this product the number thought of, and the reſult will be a ſum, from which if 3 be ſubtracted, the remainder will be decuple of the number required. If 3, therefore, be taken from the laſt ſum, and if the cipher on the right be cut off from the remainder, the other figure will indicate the number ſought.

Let the number thought of be 6, the triple of which is 18; and if unity be added it makes 19; the triple of this laſt number is 57, and if 6 be added it makes 63, from which if 3 be ſubtracted the remainder will be 60 : now if the cipher on the right be cut off, the remaining figure 6 will be the number required.

V.

Another method of telling the number any one has thought of.

Thefe operations, by which a perfon feems to guefs the thoughts of another, may be introduced very opportunely in company, when any one afferts that all amufing tricks are performed by flight of hand. The following method may be found in Ozanam, but we have here made fome additions to it. 1ft. Defire any perfon to think of a number, but that we may not fpeak in too abftract a manner, it will be beft to defire him to think of a certain number of guineas. 2d. Tell the perfon that fome one of the company lends him a fimilar fum, and requeft him to add them together, that the amount may be known. It will here be proper to name the perfon who lends him a number of guineas equal to the number thought of, and to beg the one who makes the calculation to do it with great care, as he may readilly fall into an error, efpecially the firft time. 3d. Then fay to the perfon: "I do not lend you, but give you 10, add them to the former fum." 4th. Continue in this manner: "Give the half to the poor, and retain in your memory the other half." 5th. Then add: "Return to the gentleman or lady what you borrowed, and remember that the sum lent you was exactly equal to the number you thought of." 6th. Ask the person if he knows exactly what remains; he will answer "yes:" you must then say: "and I know also the number that remains; it is equal to what I am going to conceal in my had." 7th. Put into one of your hands 5 pieces of money, and desire the person to tell how many you have got. He will reply 5; upon which open your hand and shew him the 5 pieces. You may then say: "I well knew that your result was 5; but if you had thought of a very large number, for example two or three millions, the result would have been much greater, and I should not have been able to put into my hand a number of pieces equal to the remainder." The person then supposing that the result of the calculation must be different, according to the difference of the number thought of, will imagine that it is necessary to know the last number, in order to guess the result; but this idea is false; for, in the case which we have here supposed, whatever be the number thought of, the remainder must always be 5. The reason of this is as follows: the sum, the half of which is given to the poor, is nothing else than twice the number thought of plus 10; and when the poor have received their part, there remains only the number thought of plus 5; but the number thought of is cut off when the sum borrowed is returned, and consequently there remains only 5.

It may be thence seen, that the result may be easily known, since it will be the half of the number given in the third part of the operation; for example, whatever be the number thought of, the remainder will be 36, or 25, according as 72 or 50 have been given.

Remark 1st. If this trick be performed several times successively, the number given in the third part of the operation must be always different, for if the result were several times the same, the deception might be discovered.

2d. When the five first parts of the calculation for obtaing a result are finished, it will be best not to name it at first, but to continue the operation to render it more complex, by saying for example: "Double the remainder; deduct two; add three; take the fourth part, &c. and the different steps of the calculation may be kept in mind in order to know how much the first result has been increased or diminished." This irregular process never fails to confound those who attempt to follow it.

PROB. II. *To tell two or more numbers which a person has thought of.*

I.

When each of the numbers thought of does not exceed 9; they may be easily found in the following manner.

Having made the perſon add 1 to the double of the firſt number thought of, deſire him to multiply the whole by 5, and to add to the product the ſecond number. If there be a third, make him double this firſt ſum and add 1 to it; after which deſire him to multiply the new ſum by 5, and to add to it the third number. If there be a fourth you muſt proceed in the ſame manner, deſiring him to double the preceding ſum; to add to it unity; to multiply by 5, and then to add the fourth number; and ſo on.

Then aſk the number ariſing from the addition of the laſt number thought of, and if there were two numbers ſubtract 5 from it; if there were three, 55; if there were four, 555; and ſo on; for the remainder will be compoſed of figures of which the firſt on the left will be the firſt number thought of, the next the ſecond, and ſo on.

Suppoſe the number thought of to be 3, 4, 6: by adding 1 to 6, the double of the firſt, we ſhall have 7, which being multiplied by 5, will give 35; if 4, the ſecond number thought of, be then added, we ſhall have 39, which doubled gives 78; and if we add 1, and multiply 79, the ſum, by 5, the reſult will be 395. In the laſt place, if we add 6, the number thought of, the ſum will be 401; and if 55 be deducted from it, we ſhall have for remainder 346, the figures of which 3, 4, 6, indicate in order the three numbers thought of.

II.

If one or more of the numbers thought of, be greater than 9, we must distinguish two cases: that in which the number of the numbers thought of is odd, and that in which it is even.

In the first case, ask the sum of the first and the second; of the second and third; the third and the fourth; and so on, to the last; and then the sum of the first and the last. Having written down all these sums in order, add together all those, the places of which are odd, as the first, the third, the fifth, &c.; make another sum of all those, the places of which are even, as the second, the fourth, the sixth, &c. subtract this sum from the former, and the remainder will be the double of the first number. Let us suppose, for example, that the 5 following numbers are thought of, viz. 3, 7, 13, 17, 20, which when added, two and two as above, give 10, 20, 30, 37, 23: the sum of the first, third and fifth is 63, and that of the second and fourth is 57: if 57 be subtracted from 63, the remainder 6 will be the double of the first number 3. Now if 3 be taken from 10, the first of the sums, the remainder 7 will be the second number; and by proceeding in the same manner, we may find all the rest.

In the second case, that is to say, if the number of the numbers thought of be even, you must ask and write down as above the ſum of the firſt and the ſecond; that of the ſecond and third; and ſo on, as before; but inſtead of the ſum of the firſt and the laſt, you muſt take that of the ſecond and laſt; then add together thoſe which ſtand in the even places, and form them into a new ſum apart; add alſo thoſe in the odd places, the firſt excepted, and ſubtract this ſum from the former: the remainder will be the double of the ſecond number; and if the ſecond number, thus found, be ſubtracted from the ſum of the firſt and ſecond, you will have the firſt number; if it be taken from that of the ſecond and third, it will give the third; and ſo of the reſt. Let the numbers thought of be, for example, 3, 7, 13, 17: the ſums formed as above, are 10, 20, 30, 24; the ſum of the ſecond and fourth is 44, from which if 30, the third, be ſubtracted the remainder will be 14, the double of 7 the ſecond number. The firſt therefore is 3, the third 13, and the fourth 17.

PROB. III. *A perſon having in one hand an even number of ſhillings, and in the other an odd, to tell in which hand he has the even number.*

Deſire the perſon to multiply the number in the right hand by any even number whatever, ſuch as 2; and that in the left by an odd number, as 3; then bid him add together the two products, and if the whole ſum be odd, the even number of ſhillings will be in the right hand, and the odd number in the left; if the sum be even, the contrary will be the case.

Let us suppose, for example, that the person has 8 shillings in his right hand, and 7 in his left; 8 multiplied by 2 gives 16, and 7 multiplied by 3 gives 21: the sum of which, 37, is an odd number.

If the number in the right hand were 9, and that in the left 8, we should have $9 \times 2 = 18$, and $8 \times 3 = 24$; the sum of which two products is 42, an even number.

PROB. IV. *A person having in one hand a piece of gold, and in the other a piece of silver; to tell in which hand he has the gold, and in which the silver.*

For this purpose, some value, represented by an even number, such as 8, must be assigned to the gold, and a value represented by an odd number, such as 3, must be assigned to the silver; after which, you may proceed exactly in the same manner as in the preceding example.

1st. To conceal the artifice better, it will be sufficient to ask whether the sum of the two products can be halved without a remainder; for in that case the total will be even, and in the contrary case odd.

2d. It may be readily seen that the pieces, instead of being in the two hands of the same person, may be supposed to be in the hands of two persons, one of whom has the even number, or piece of gold, and the other the odd number, or piece of ſilver. The ſame operations may then be performed in regard to theſe two perſons, as are performed in regard to the two hands of the ſame perſon, calling the one privately the right, and the other the left.

PROB. V. *The game of the ring.*

This game is nothing elſe, than an application of one of the methods employed to tell ſeveral numbers thought of, and ought to be performed in a company not exceeding 9, in order that it may be leſs complex. Deſire any one of the company to take a ring, and to put it on any joint of whatever finger he may think proper. The queſtion then is, to tell what perſon has the ring, and on what hand, what finger, and what joint.

For this purpoſe, you muſt call the firſt perſon 1, the ſecond 2, the third 3, and ſo on. You muſt alſo denote the 10 fingers of the two hands, by the following numbers of the natural progreſſion 1, 2, 3, 4, 5, &c. beginning at the thumb of the right, and ending at that of the left, that by this order the number of the finger may at the ſame time indicate the hand. In the laſt place, the joints muſt be denoted by 1, 2, 3, beginning at the points of the fingers.

To render the ſolution of this problem more explicit, let us ſuppoſe that the fourth perſon in the company has the ring, on the ſixth finger, that is to ſay, on the little finger of the left hand, and on the ſecond joint of that finger.

Deſire ſome one to double the number expreſſing the perſon which, in this caſe will give 8; bid him add 5 to this double, and multiply the ſum by 5, which will make 65; then tell him to add to this product the number denoting the finger, that is to ſay 6, by which means you will have 71; and, in the laſt place, deſire him to multiply the laſt number by 10, and to add to the product the number of the joint 2. The laſt reſult will be 712: if from this number you deduct 250, the remainder will be 462; the firſt figure of which, on the left, will denote the perſon; the next the finger, and conſequently the hand, and the laſt the joint.

It muſt here be obſerved, that when the laſt reſult contains a cipher, which would have happened in the preſent example had the number of the finger been 10, you muſt privately ſubtract, from the figure preceding the cipher, and aſſign the value of ten to the cipher itſelf.

The ſame formula, as may be readily conceived, will anſwer for all caſes whatever.

PROB. VI. *To gueſs the number of ſpots on any card which a perſon has drawn from a whole pack.*

Take a whole pack, conſiſting of 52 cards, and deſire ſome perſon in company to draw out any card, at pleasure, without shewing it. Having assigned to the different cards their usual value, according to their spots, call the knave 11, the queen 12, and the king 13. Then add the spots of the first card to those of the second; the last sum to the third; and so on, always rejecting 13, and keeping the remainder to add to the following card. It may be readily seen that it is needless to reckon the kings which are counted 13. If any spots remain at the last card, you must subtract them from 13, and the remainder will indicate the card that has been drawn: if 12 remains, it has been an ace; but if nothing remains, it has been a king.

Demonstration.

Since a complete pack contains 13 cards of each suit, the values of which are 1, 2, 3, &c. as far as 13, the sum of all the spots of each of the different suits, will be 7 times 13 (21), which is a multiple of 13; consequently the quadruple is also a multiple of 13: if we add therefore, the spots of all the cards, always rejecting 13, the remainder at last must be 0. Hence it is evident that if a card, the spots of which are less than 13, be drawn, the difference between its spots and 13 will be what is wanting to complete the number. If at the end then, instead of attaining to 13, we attain only to 10, for example, it is plain that the card wanting is a 3; and if we attain exactly to 13, the card misſing muſt be equivalent to 13; that is, it muſt be a king.

PROB. VII. *A perſon having a certain number of counters in each hand; to find how many he has altogether.*

Deſire the perſon to convey 4, for example, from the one hand to the other; and then aſk him how many times the leſs number is contained in the greater? Let us ſuppoſe that he ſays the one is the triple of the other; in this caſe multiply 4, the number of counters conveyed from one hand into the other, by 3, and add the ſame number, which will make 16. In the laſt place, from the ſame number 3, ſubtract unity, and if you divide 16 by 2, the remainder, the quotient 8 will be the number contained in each hand; and conſequently the whole number is 16.

Let us now ſuppoſe, that when 4 counters are conveyed from one hand to the other, the leſs number is contained in the greater $2\frac{1}{3}$ times: in this caſe, we muſt, as before, multiply 4 by $2\frac{1}{3}$, which will give $9\frac{1}{3}$; to which if 4 be added, we ſhall have $13\frac{1}{3}$, or $\frac{40}{3}$. Then if unity be taken from $2\frac{1}{3}$, the remainder will be $1\frac{1}{3}$ or $\frac{4}{3}$; by which if $\frac{40}{3}$ be divided, the quotient 10 will be the number of counters in each hand, as may be eaſily proved on trial.

PROB. VIII. *Several cards being given; to tell which of them a perſon has thought of.*

Deſire the perſon to remember the card, and its place in the pack, counting from the bottom. Then take the cards, and in a dexterous manner, ſo as not to be perceived, convey a certain number of them from the top to the bottom; and ſubtract them in your mind from the pack, with the number of which you are acquainted. If the pack, for example, conſiſts of 52 cards, and you have conveyed 8 to the bottom, tell the perſon that the card he has thought of will be the forty-fourth, reckoning from the card the place of which he is going to name. Thus, if he ſays it is the ninth, you go on counting 9, 10, 11, &c. and the card he thought of will be exactly the forty-fourth, as you announced.

PROB. IX. *Having ſpread out on the table 20 cards, arranged two and two, and deſired one or more perſons to think of two, provided they lie cloſe to each other, to tell which cards they have thought of.*

You muſt retain in your memory the four following words, with the arrangement of the letters which compoſe them:

m	*i*	*s*	*a*	*i*
t	*a*	*t*	*l*	*o*
n	*e*	*m*	*o*	*n*
v	*e*	*s*	*y*	*l*

Collect all the cards into the left hand, two by two, as they lay on the table, and then place them, one by one, in the same order as the preceding letters, taking care to place the two first as the two *m*; the two next as the two *i*, the two following as the two *s*, and so on.

Ask each person in which horizontal row his two cards are. If he says they are both in the same row, for example the third, they will be pointed out by the letters *n* and *n*, contained in that row; if they are in two different rows, as the first and last, the letters *s* and *s* will indicate the place which they occupy.

PROB. X. *To make all the cards of the same kind to be found together, however often the pack may have been cut.*

Have in readiness a pack, all the cards of which are arranged in successive order; that is to say, if it consist of 52 cards, every 13 must be regularly arranged, without a duplicate of any one of them. After they have been cut as many times as a person may choose, form them into 13 heaps of 4 cards each, with the coloured faces downwards. When this is done, the 4 kings, the 4 queens, the 4 knaves, and so on must necessarily be together.

PROB. XI. *The four indivisible kings.*

Take four kings, and place between the third and fourth any two common cards whatever, which must be neatly concealed; then shew the four kings, and place the six cards at the bottom of the pack; take one of the kings, and lay it on the top, and put one of the common cards into the pack nearly about the middle; do the same with the other, and then show that there is still one king at the bottom: desire any one to cut the pack, and as three of the kings were left at the bottom, the four will therefore be found together in the middle of the pack.

PROB. XII. *Two heaps of cards being displayed on a table, to write down on a piece of paper that heap which a person will choose.*

Place in a heap 2 or 3 sevens; and in another 7 cards. Write on a bit of paper the word *seven*, and invert it, that what you have written may be concealed: then desire any one to choose, and when that is done, turn up the heap chosen, and prove the truth of your prediction by shewing what you wrote; but you must take care to shew only the heap which has been chosen.

PROB. XIII. *Several cards being presented in succession to several persons, that they may each choose one at pleasure; to guess that which each has thought of.*

Shew as many cards to each person as there are persons to choose; that is to say, 3 to each, if there are 3 persons. When the first has thought of one, lay aside the three cards in which he has made his choice. Present the same number to the second person, to think of one, and lay aside the three cards in the like manner. Having done the same in regard to the third person, arrange all these cards in three rows, with their faces turned downwards, and then put them together in order. If you take the 3 first, and present them successively to the different persons, and do the same thing with the others, you may easily guess the cards, by observing that the card thought of by each person will have the same place among the cards as the person has in regard to the other two ; that is to say, the card thought of by the first person, will be first of that packet in which he discovered it ; that thought of by the second, will be the second in the packet, where he recognized it ; and that of the third, will be the last and in the last packet.

The operation is exactly the same when the number of persons is greater. If, instead of 3, there are 4, or 5 persons, four or five cards must be presented to each.

PROB. XIV. *Three cards being presented to three persons ; to guess that which each has chosen.*

As it is necessary that the cards presented should be distinguished, we shall call the first A, the second B, and the third C. Let the persons, whom we shall distinguish by first, second, and third, choose privately which ever of the cards they think proper, and when they have made their choice, which is susceptible of 6 varieties, give the first person 12 counters, the second 24, and the third 36: then desire the first person to add together the half of the counters of the person who has chosen the card A; the third of those of the person who has chosen B; and the fourth part of those of the person who has chosen C; and ask the sum, which must be either 23 or 24; 25 or 27; 28 or 29, as in the following table:

First.	Second.	Third.	Sums.
12	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

This table shews, that if the sum is 25, for example, the first person must have chosen the card B, the second the card A, and the third the card C; and that, if it be 28, the first person must have chosen the card B, the second the card C, and the third the card A; and so of the rest.

PROB. XV. *To tell the number of spots on all the bottom cards of several heaps, arranged on a table.*

Arrange each heap of cards in such a manner, that the spots on the bottom one, added to the cards above it, may always amount to 12; continue to make as many heaps as poſſible, in the manner above preſcribed, and place the remaining cards on one ſide. Then ſeparate in your mind four heaps, and multiply the heaps which remain, after theſe are deducted, by 13; this product, added to the number of cards, will be that of the ſpots required. We ſhall give the ſolution of this problem by an analyſis in another place.

PROB. XVI. *To name all the cards of a pack.*

Have a complete pack of 52 cards, and arrange them according to the order of the following words, which you muſt retain in your memory:

Unus	*quinque*	*novem*	*famulus*	*ſex*	*quatuor*	*duo*
Ace	five	nine	knave	ſix	four	two

Rex	*ſeptem*	*octo*	*fæmina*	*trina*	*decem*
King	ſeven	eight	queen	three	ten

Beſides this firſt order, you muſt arrange them alſo according to the order of the colours, ſpades, hearts, clubs, and diamonds; ſo that the 52 cards may be diſpoſed as follows:

Order of the Cards.

1	Ace of ſpades	27	Ace of clubs
2	Five of hearts	28	Five of diamonds
3	Nine of clubs	29	Nine of ſpades
4	Knave of diamonds	30	Knave of hearts
5	Six of ſpades	31	Six of clubs
6	Four of hearts	32	Four of diamonds
7	Two of clubs	33	Two of ſpades
8	King of diamonds	34	King of hearts
9	Seven of ſpades	35	Seven of clubs
10	Eight of hearts	36	Eight of diamonds
11	Queen of clubs	37	Queen of ſpades
12	Three of diamonds	38	Three of hearts
13	Ten of ſpades	39	Ten of clubs
14	Ace of hearts	40	Ace of diamonds
15	Five of clubs	41	Five of ſpades
16	Nine of diamonds	42	Nine of hearts
17	Knave of ſpades	43	Knave of clubs
18	Six of hearts	44	Six of diamonds
19	Four of clubs	45	Four of ſpades
20	Two of diamonds	46	Two of hearts
21	King of ſpades	47	King of clubs
22	Seven of hearts	48	Seven of diamonds
23	Eight of clubs	49	Eight of ſpades
24	Queen of diamonds	50	Queen of hearts
25	Three of ſpades	51	Three of clubs
26	Ten of hearts	52	Ten of diamonds

This order is of such a nature, that by knowing any one of the 52 cards, that which follows it may be also known.

Thus, for example, if it were required to know what card follows the king of spades, it will be sufficient to recollect that *septem*, in the two latin lines above given, which follows that of *rex*, denotes that it is a seven; and as the colour which follows the spades is hearts, it is the seven of hearts, and so of the rest.

Every thing being thus arranged, having retained in your memory the above words, and the order of the colours, desire any person to cut the pack as many times as he chooses; for it will be easy to name all the cards in order, provided you have found means, by some dexterous manœuvre, to observe that one which is at the top of the pack.

The same arrangement of the cards may be employed for various amusements.

1st. To make a person believe that you can distinguish the cards by their smell.

The pack being disposed in the above order, present it to any one, that he may choose a card at pleasure, open the pack at the place where it has been drawn out, and dexterously observe that which precedes it, by seeming to smell the place from which it was taken. It will then be very easy to name it, as it can be only that which follows in the order already indicated.

2d. A pack of cards being divided into two parts; to discover whether the number in each be odd or even.

First, find out whether the last card in the pack be black or red; then, on the pack being cut into two parts, if the card found at the bottom of the upper division is of the same colour as that at the bottom of the pack, the two parts which have been separated, contain each an even number; on the other hand, if it be of a different colour, they contain each an odd number.

3d. To tell the number of spots on several cards which any person has chosen.

Having presented the pack, that the person may choose several succeeding cards at pleasure, privately observe the card which is above those he has chosen, and how many he has drawn from the pack; it will then be easy to count how many spots they ought to contain.

For example, if the observed card be a nine, and four cards have been drawn, it may readily be seen that those drawn must be a knave, equivalent to 10 spots; a six, a four and a two. You may then announce that the cards, in the persons hand, contain 22 spots.

PROB. XVII. *Having desired a person to draw four cards from a pack, and to think of one of them; to tell the one he has thought of.*

Suffer the person to draw four cards from the pack at pleasure, and desire him to think of one of them; then take these four cards back, and place two of them at the top and two at the bottom of the pack, in a dexterous manner, so as not to be perceived: under the two last, place any four cards whatever; then display the lower part of the pack on the table, shewing only 8 or 10 cards, and ask the person whether the one he thought of be among them; if he says no, you may be sure that it is one of the two which you put at the top of the pack; in that case you must transfer them to the bottom, and then, shewing the bottom of the pack, say: "Is not this your card?" If he replies "no," turn aside that card with your third finger, which you must have previously moistened, and desire him to draw out his card himself from the bottom of the pack.

If the person should say that the card he thought of is among the first shewn to him, dexterously remove the four cards put at the bottom of the pack, in order that the two, one of which is the card he thought of, may be the lowermost of the pack, and you may then either shew him his card or make him draw it out himself, as above explained.

PROB. XVIII. *Three things being privately distributed to three persons, to guess that which each has got.*

Let the three things be a ring, a shilling, and a glove. Call the ring A, the shilling E, and the glove I; and in your own mind distinguish the persons by calling them first, second, and third. Then take 24 counters, and give one of them to the first person, two to the second, and three to the third. Place the remaining 18 on the table, and then retire, that the three persons may distribute among themselves the things proposed, without your observing them. When the distribution has been made, desire the person who has the ring to take from the 18 remaining counters, as many as he has already; the one who has the shilling to take twice as many as he has already, and the person who has the glove to take four times as many. By these different combinations the counters left can be only 1, 2, 3, 5, 6, or 7. When this is done, you may return, and by the number of counters left you can discover what thing each has got, by employing the following words:

1	2	3	5	6	7
Par fer	*César*	*jadis*	*devint*	*si grand*	*prince.*

To make use of these words, you must recollect what has been already said, viz. that the number of the counters which remain, can be only 1, 2, 3, 5, 6 or 7, and never 4; you must obſerve alſo, that each ſyllable contains one of the vowels which we have made to repreſent the three things propoſed, and that the above line muſt be conſidered as conſiſting only of ſix words: the firſt ſyllable of each word muſt alſo be ſuppoſed to repreſent the firſt perſon, and the ſecond ſyllable the ſecond perſon. This being comprehended, if there remains only one counter you muſt employ the firſt word, or rather the two firſt ſyllables *par ſer*, the firſt of which, that containing A, ſhews that the firſt perſon has the ring repreſented by A; and the ſecond ſyllable, that containing E, ſhews that the ſecond perſon has the ſhilling, repreſented by E; from which you may eaſily conclude that the third perſon has the glove. If two counters remain, you muſt take the ſecond word *Céſar*, the firſt ſyllable of which, containing E, will ſhew that the firſt perſon has the ſhilling, repreſented by E; and the ſecond ſyllable, containing A, will indicate that the ſecond perſon has the ring, repreſented by A: you may then eaſily conclude that the third perſon has the glove.

PROB. XIX. *To tell, by inſpecting a watch, at what hour a perſon has reſolved to riſe next morning.*

1ſt. When the perſon has thought of an hour, bid him touch ſome other hour on the dial-plate, and then desire him to add 12 to it privately in his own mind, which will form a certain number.

2d. Then desire him to proceed backwards, and to count the above number, beginning with the hour which he thought of.

Let the hour thought of, for example, be 8, and that touched be 3: as 12 added to 3 makes 15, desire the person to count that number, in a retrograde order from the hour touched, beginning with 8 the hour thought of; counting 8 on the hour 3, 9 on 2, 10 on 1, and so on, by which means 15 will fall upon the hour of 8.

The person will be surprised to find that he has fallen on the hour he thought of.

PROB. XX. *Two persons agree to take alternately numbers less than a given number, for example* 11, *and to add them together till one of them has reached a certain sum, such as* 100; *by what means can one of them infallibly attain to that number before the other?*

The whole artifice of this problem consists in immediately making choice of the numbers 1, 12, 23, 34, and so on, or of a series which continually increases by 11, up to 100.

Let us suppose that the first person, who knows the game, makes choice of 1; it is evident that his adversary, as he must count less than 11, can at most reach 11 by adding 10 to it. The first will then take 1, which will make 12; and, whatever number the second may add, the first will certainly win, provided he continually adds the number which forms the complement of that of his adversary to 11: that is to say, if the latter takes 8, he must take 3; if 9, he must take 2, and so on. By following this method, he will infallibly attain to 89; and it will then be impossible for the second to prevent him from getting first to 100; for whatever number the second takes, he can attain only to 99; after which the first may say: "and 1 makes 100." If the second takes 1, after 89, it would make 90; and his adversary would finish by saying: "and 10 make 100."

It is evident that, when two persons are equally well acquainted with the game, he who begins must necessarily win.

Prob. xxi. *Sixteen counters being disposed in two rows, to find that which a person has thought of.*

The counters being arranged as follow, desire the person to think of one, and to observe well in which row it is:

A	B	C	D	E	F	H	I
o	o	o	o	o	o	o	*
o	o	o	o	*	o	o	o
o	o	*	o	o	o	o	o
o	o	o	o	o	o	o	o
*	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o

Let us ſuppoſe that the counter thought of is in the row A: take up the whole row in the order in which it now ſtands, and diſpoſe it in two rows C and D, in ſuch a manner, that the firſt counter of the row A may be the firſt of the row C; the ſecond of the row A, the firſt of the row D; and ſo on, transferring the 16 counters from A and B, to C and D. This being done, again aſk in which of the vertical rows the counter thought of ſtands. We ſhall ſuppoſe it to be in C: remove that row as well as D, obſerving the ſame method as before; and continue in this manner until the counter thought of becomes the firſt of the row I. If you then aſk in which row it is, it may be immediately known, becauſe after the laſt operation it will be the firſt in the row ſaid to contain it; and as each row has a diſtinguiſhing character or ſign, you may cauſe them all to be mixed with each other, and ſtill be able to diſcover it by the ſign you have remarked.

Inſtead of 16 counters, 16 cards may be employed. After you have diſcovered the one thought of, you may cauſe them to be mixed, which will conceal the artifice.

If a greater number of counters or cards be employed, diſpoſed in two vertical rows, the counter or card thought of will not be at the top of the row after the laſt tranſpoſition: if there are 32 counters or cards, 4 tranſpoſitions will be neceſſary; if 64, there muſt be 5; and ſo on.

PROB. XXII. *A certain number of cards being ſhewn to a perſon, to gueſs that which he has thought of.*

To perform this trick, the number of the cards muſt be diviſible by 3; and to do it with more convenience, the number muſt be odd.

The firſt condition, at leaſt, being ſuppoſed, the cards muſt be diſpoſed in three heaps with their faces turned upwards. Having then aſked the perſon in which heap is the card thought of, place the heaps one above the other, in ſuch a manner that the one containing the card thought of may be in the middle. Arrange the cards again in three heaps, and having aſked in which of them is the card thought of, repeat the operation as before. Arrange them a third time in three heaps, and having once more aſked the ſame queſtion, form them all into one heap, that containing the card thought of being in the middle. The card thought of muſt then neceſſarily be the middle one, that is to ſay, if 15 cards have been employed, it will be the eighth from the top; if 21, the eleventh; if 27, the fourteenth; and ſo on. When the number of the cards is 24, it will be the twelfth, &c.

PROB. XXIII. *To arrange 30 criminals in ſuch a manner, that by counting them in ſucceſſion, always beginning again at the firſt, and rejecting every ninth perſon, 15 of them may be ſaved.*

Arrange the criminals according to the order of the vowels, in the following Latin verſe,

4 5 2 1 3 1 1 2 2 3 1 2 2 1
Populeam virgam mater regina ferebat.

Becauſe *o* is the fourth in the order of the vowels, you muſt begin by four of thoſe whom you wiſh to ſave; next to theſe place five of thoſe whom you wiſh to puniſh; and ſo on alternately, according to the figures which ſtand over the vowels of the above verſe.

In a company conſiſting of ſeveral perſons, the following game may be introduced by way of amuſement.

We ſhall ſuppoſe that there are 13 ladies in the company: in that caſe, provide 12 noſegays, and in order to mortify one of them, without ſhewing any appearance of partiality, announce that you mean to let chance decide which of them is to go without one. For this purpose, make the 13 ladies ſtand up in a ring, allowing them to place themſelves as they pleaſe; and diſtribute to them the 12 noſegays, counting them from 1 to 9, and making the ninth retire from the ring and carry with her a noſegay. It will be found that the eleventh, reckoning from the one by whom you began, will remain the laſt; and conſequently will have no ſhare in the diſtribution.

The following table will ſhew the perſon, before her whom you wiſh to exclude, with whom you muſt begin to count 9, ſuppoſing always that the number of the noſegays is leſs by 1 than that of the perſons.

For 13 perſons, the 11th before.

12	2d.
11	5th.
10	7th.
9	8th.
8	8th.
7	7th.
6	5th.
5	3d.
4	3d.
3	2d.
2	1ſt.

PROB. XXIV. *A man has a wolf, a goat, and a cabbage, to carry over a river; but, as he is obliged to transport them one by one, in what manner is this to be done, that the wolf may not be left with the goat, nor the goat with the cabbage?*

He must first carry over the goat, and then return for the wolf; when he carries over the wolf, he must take back with him the goat, which he must leave, in order to carry over the cabbage; he may then return, and carry over the goat. By these means the wolf will never be left with the goat, nor the goat with the cabbage, but when the boatman is present.

PROB. XXV. *In what manner can counters be disposed in the eight external cells of a square, so that there may always be 9 in each row, and yet the whole number shall vary from 20 to 32?*

This problem may be proposed in the following manner: A wine-merchant caused 32 casks of choice wine to be deposited in his cellar, giving orders to his clerk to arrange them in the annexed figure, so that each external row should contain 9.

1st order.

1	7	1
7		7
1	7	1

The clerk, however, took away 12 of them, at three different times; that is, 4 each time; yet when the merchant went into the cellar, after each theft had been committed, the clerk always made him count 9, in each row. How was this possible?

This problem may be easily solved by inspecting the following figures:

2d order.

2	5	2
5		5
2	5	2

3d order.

3	3	3
3		3
3	3	3

4th order.

4	1	4
1		1
4	1	4

PROB. XXVI. *To diſtribute among 3 perſons, 21 caſks of wine, 7 of them full, 7 of them empty, and 7 of them half full, ſo that each of them ſhall have the ſame quantity of wine, and the ſame number of caſks.*

This problem admits of two ſolutions, which may be clearly comprehended by means of the two following tables :

	Perſons.	full caſks.	empty.	half full.
I.	1ſt	2	2	3
	2d	2	2	3
	3d	3	3	1

	Perſons.	full caſks.	empty.	half full.
II.	1ſt	3	3	1
	2d	3	3	[illegible]
	3d	1	1	5

PROB. XXVII. *A ſchoolmaſter, to amuſe his ſcholars, ſhewed them a number, which he ſaid was the ſum of 6 rows, each conſiſting of 4 figures; he then deſired them to write down 3 rows of figures, to which he would add 3 more, and aſſured them that the ſum of the whole ſhould be equal to the number he ſhewed them.*

To ſolve this problem, multiply in your own mind 9999 by 3; and the product 29997 will be the number which the ſchoolmaſter ſhewed to his ſcholars.

Rows of the scholars,	7285 5829 3456
Rows of the master,	2714 4170 6543
Total	29997

It may be here seen that each figure set down by the master is the complement to 9 of that set down by the scholars; and consequently the sum, though written down before-hand, must be exact.

PROB. XXVIII. *Having desired any person to multiply, for example, one of the three following numbers, by any figure at pleasure, and to tell you the product, after suppressing one figure of it, and even changing the order of the rest, to guess the figure that has been suppressed.*

Let the three given numbers be

364851
234765
823644

If we suppose the person to multiply the third number by 6, the product of which will be 4941864, whatever figure be effaced, it may be easily discovered by that wanting to complete the product, as the sum of its figures must necessarily be a multiple of 9. If the 6, for example, be suppressed, the sum will not be a multiple of 9; for it amounts only to 30: as 6 therefore is wanting to 30 to make it a multiple of 9, you may boldly assert that 6 has been suppressed.

As the sum of the figures would still be a multiple of 9, if a cypher were suppressed, and as it would consequently have no need of being complete, you must make it a condition of the problem that the person shall suppress only one significant or effective figure; and if you find that the sum has no need of being completed, you may conclude that the figure suppressed has been a 9.

A mountebank, to give the greater air of the marvellous to this sport, pretended to discover by the smell what figure had been suppressed: but it may easily be supposed that while he pretended to smell the figures, he privately added them together, so as to discover their sum.

There is another method of guessing the suppressed figure, even when the person has been allowed to write down the sum to be multiplied himself; but in this case you must stipulate to have permission to add any one figure you choose: you must observe what figure is wanting to complete the sum, and set down that figure; if nothing is wanting, you may add 0, or 9.

PROB. XXIX. *A person having made choice of two numbers, and multiplied them together, to tell the product, provided you know only the last figure of it.*

Have in readineſs a ſmall bag with two diviſions, and put into one of them 12 ſquare bits of card, each inſcribed with the number 73; and into the ſecond 9 other pieces, inſcribed with the terms of the arithmetical progreſſion, 3, 6, 9, 12, 15, 18, 21, 24, 27.

Preſent that aperture of the bag which contains the numbers 73, and deſire the perſon to draw out one; then dexterouſly change the ſide of the bag, and having deſired another perſon to draw any number from the ſecond diviſion, bid him multiply the number he has taken by that drawn out by the firſt perſon: the product will neceſſarily be one of the nine numbers 219, 438, 657, 876, 1095, 1314, 1533, 1752, 1971. You may then eaſily tell the product of the multiplication, if you know only the laſt figure of it.

It muſt here be obſerved that this recreation requires a good memory; as it will be neceſſary to know by heart the above nine products: The following, founded on the ſame principle, is much eaſier.

PROB. XXX. *A perſon having choſen two numbers, and divided the greater by the leſs, to tell the quotient; that is to ſay, how many times the leſs is contained in the greater.*

Put into the firſt diviſion of the bag the nine numbers 219, 438, 657, 876, 1095, 1314, 1533, 1752, 1971; and into the ſecond, the cards inſcribed with the number 73: deſire the perſon to draw a number from each diviſion, and to divide the one by the other; then aſk him to tell you the laſt figure of the greater of the numbers, as it will enable you to diſcover which of the nine numbers of the above arithmetical progreſſion is the quotient: thus, if it be a 9, the number 3 is the quotient; if it be an 8, the quotient is the number 6; and ſo on: for the quotients 3, 6, 9, 12, 15, 18, 21, 24, and 27, will be in the ratio of the figures 1, 2, 3, 4, 5, 6, 7, 8 and 9, with which the greater number muſt neceſſarily terminate.

POLITICAL ARITHMETIC.

SINCE politicians have acquired jufter ideas refpecting what conftitutes the real ftrength of ftates, various refearches have been made in regard to the number of the inhabitants in different countries, in order to afcertain their population. Befides, as almoft all governments have been under the neceffity of making loans, for the moft part on annuities, they have naturally been induced to examine according to what progreffion mankind die, that the intereft of thefe loans may be proportioned to the probability of the annuities becoming extinct. Thefe calculations have been diftinguifhed by the name of political arithmetic; and, as it exhibits feveral curious facts, whether confidered in a political or philofophical point of view, we have thought it our duty to give it a place here, to amufe and inftruct our readers.

§ 1. *Of the proportion between the males and the females.*

Many people imagine that the number of the females born exceeds that of the males; but it has long ſince been proved that the contrary is the caſe. More boys than girls are born every year; and ſince the year 1631, a ſmall interval excepted, we have a regiſter of births in regard to ſex, and it has never been obſerved that the number of the females born ever even equalled that of the males. It is found, by taking a mean or average term in a great number of years, that the number of the males born is to that of the females, as 18 to 17. This proportion is nearly that which prevails throughout all France; but, to whatever reaſon owing, it ſeems at Paris to be as 27 to 26.

This kind of phenomenon is obſerved not only in England and France; but in every other country. We may be convinced of the truth of it by inſpecting the calendars, which, at the commencement of every year, give a table of the births that have taken place in moſt of the capital cities of Europe; it will there be ſeen, that the number of the males born always exceeds that of the females; and conſequently it may be conſidered as a general law of nature.

We may here obſerve a ſtriking inſtance of the wiſdom of Providence, which has thus provided for the preservation of the human race. Men, in consequence of the active life for which they are naturally destined by their strength and their courage, are exposed to more dangers than the female sex; war, long sea voyages, occupations laborious or prejudicial to health, and dissipation, carry off great numbers of the males; and it thence results, that if the number born of the latter did not exceed that of the females, the males would rapidly decrease, and soon become extinct.

§ II. *Of the mortality of the human race, according to the different ages.*

In this respect there is apparently a considerable difference between large towns and the country; but this arises from the women in towns rarely suckling their own children; and consequently the greater part of their children being put out to nurse in the country, as it is in the period of childhood that the greatest mortality prevails, it is most apparent in the country. To make an exact calculation, it ought to be founded on the deaths which happen in the towns, as well as in the country; and this M. Dupré de St. Maur endeavoured to do, by comparing the registers of three parishes in Paris, and twelve in the country.

According to the observations of this author, in 23994 deaths, 6454 of them were those of children not a year old. After carrying his researches on this ſubject as far as poſſible, he concludes that, of 24000 children born, the numbers who attain to different ages, are as follow:

Ages.	Number.
2 years	17540
3	15162
4	14177
5	13477
6	12968
7	12562
8	12255
9	12015
10	11861
15	11405
20	10909
25	10259
30	9544
35	8770
40	7929
45	7008
50	6197
55	5375
60	4564
65	3450
70	2544
75	1507
80	807
85	291

Ages.	Number.
90	103
91	71
92	63
93	47
94	40
95	33
96	23
97	18
98	16
99	8
100	6 or 7.

Such then is the condition of the human ſpecies, that, of 24000 children born, ſcarcely one half of them attain to the age of 9; and that two thirds are in their grave before the age of 40. About a ſixth only remain at the expiration of 62 years; a tenth after 70 years; a hundredth part after 86; about a thouſandth part attain to the age of 96; and ſix or ſeven individuals to that of 100.

By means of this table we may aſcertain, pretty nearly, what probability there is of a new-born child attaining to a certain age; for this probability muſt be to the contrary probability, as the number of thoſe who attain to that age is to the number of thoſe who die before it.

For example, as 4564 ſtands oppoſite to 60, it indicates that as, of 24000 children born, there remain no more than the above number of individuals at the end of 60 years, 19436 muſt have died; the probability therefore of a child attaining to the age of 60, is to the probability of its not attaining to it, as 4564 is to 19436. In this caſe, the proportion of thoſe living to thoſe dead, is nearly as 1 to 4; from which we may conclude, that the chance is 4 to 1 that a new-born child will not attain to the age of 60.

If the probability of a perſon, of any determinate age, living to another age, be required, for example, that of a child of eight years of age attaining to the age of 60; we muſt compare the number of thoſe who attain to the age of 60, with that of thoſe who attain to the age of 8; and the difference 7691 will give the number of thoſe who die between theſe two periods. We ſhall then have this analogy: as 4564 is to 7691, ſo is the probability, that a child of 8 years will attain to the age of 60, to the probability of its not attaining to it. If 7691 be divided by 4564, it will be found that the former contains the latter nearly twice; and we may therefore ſay, that the chance is almoſt 2 to 1, that a child of 8 years of age will not live to that of 60.

§ III. *Of the number of men of different ages in a given number.*

It may be deduced from the preceding obſervations, that, when the inhabitants of a country amount to a million, the number of thofe of the different ages will be as follows:

Between	0	and 1 year complete	38740
	1	5	119460
	5	10	99230
	10	15	94530
	15	20	88673
	20	25	82380
	25	30	77650
	30	35	71665
	35	40	64205
	40	45	57230
	45	50	50605
	50	55	43940
	55	0	37110
	60	65	28690
	65	70	21305
	70	75	13195
	75	80	7065
	80	85	2880
	85	90	1025
	90	95	335
	95	100	82
Above	100 years		3 or 4.

Thus, in a country peopled with a million of inhabitants, there are about 536350 between the age of 15 and 60; and as nearly one half of them are men, confequently this number of inhabitants could, on an emergency, furnifh 250 thoufand men capable of bearing arms, even if an allowance be made for the ſick, lame, &c. who may be ſuppoſed to be among that number.

§ IV. *Of the proportion of the births and deaths to the whole number of the inhabitants of a country—The conſequences thence deduced.*

As it would be difficult to number the inhabitants of a country, and much more ſo to repeat the enumeration as often as it might be neceſſary to aſcertain the population, means have been deviſed for accompliſhing the ſame object, by determining the proportion which the births and deaths bear to the whole number of the inhabitants; for, as regiſters of births and deaths are regularly kept in all the civilized countries of Europe, we may judge, by comparing them, whether the population has increaſed or decreaſed; and, in the latter caſe, can examine the cauſes which have produced the diminution.

The proportion of the births to the whole population in three generalities of France, which differ from each other as much by the nature as the form of the ſoil, give the mean ratio of 1 to $25\frac{1}{2}$, without including the large towns; ſo that in this country we may reckon 51 inhabitants for two births.

But as, in towns of any magnitude, there are ſeveral claſſes of citizens who ſpend their lives in celibacy, and who contribute either nothing or very little to the population, it is evident that the proportion between the births and the effective inhabitants, must be more considerable. It has been ascertained by various comparisons that the proportion, nearest the truth, is that of 1 to 28; and it is this ratio which ought to be employed, in order to deduce from the births, in a large city, the number of the inhabitants.

But there is reason to believe that, in regard to cities of the first class, or capitals, such as London, Paris, Amsterdam, &c. which are frequented by multitudes of strangers, invited thither either by pleasure or business, and where great luxury prevails, which increases the number of those who live in voluntary celibacy, the above proportion must be raised, and carried at least to that of 1 to 30 or 31.

§ V. *Of some other proportions, in regard to the inhabitants of a country.*

We may deduce, by approximation, from the observations of various authors in England, France, Holland, and Germany:

1st. That the number of the inhabitants of a country, is to that of the families, as 1000 to 222½; so that 2000 inhabitants give, in general, 445 families.

2d. That the number of the male children, exceeds that of the female; and that this excess continues for more than 14 years, according to the proportion of nearly 30 to 29. After 14 years however, the number of the females exceeds that of the males, in the proportion of about 19 to 18, on account of the considerable decrease of the males by war, navigation, laborious occupations, and intemperance..

3d. That the number of the marriages, is annually to that of the inhabitants, as 1 to 112.

4th. That the proportion of married men or widowers, to the number of wives or widows, is nearly as 125 to 140; and the whole number of this class, is to the whole of the inhabitants, as 59 to 126.

5th. That the number of widowers is to that of widows, nearly as 1 to 3. This at least is the proportion deduced, from the enumeration of the people made in Holland and in England. And it ought not to appear astonishing, if it be considered that most men marry at a later period of life than the women, and that their laborious occupations, the maritime and land wars in which they are engaged, and the diversity of the climates which they frequent for the sake of commerce, must increase the number of the widows in the bills of mortality.

6th. That, admitting the above proportion of widowers and widows, it follows that, among 651 inhabitants, there are 118 married couples; from 7 to 3 widowers; from 21 to 22 widows; and the rest are composed of children, persons in a state of celibacy, domestics, and passengers:

7th. That 1870 married couplies produce annually 357 children; for a town having 10000 inhabitants would contain that number of married couples, and give annually 357 births; from which it is concluded, that 5 married couples, of all ages, give annually, one with another, one birth.

8th. That the number of servants is to the whole number of inhabitants, nearly as 136 to 1535; which is a little more than the eleventh part. The number of male domestics is nearly equal to that of the female; being in the proportion of 67 to 69; but it is probable that, in large cities, where great luxury prevails, the proportion must be different.

The above observations will enable us to solve the following problem, and may serve to facilitate the solution of others relating to the same subject.

PROB. I. *The age of a man being given, suppose that of 30 years, what probability is there that he will be living at the end of a determinate number of years, for example 15?*

To resolve this problem, seek in the table of the second section for the given age of the person, viz. 30, and observe the number opposite to it, which is

9544; then find in the same table the number opposite to 45; which is 7008, and make the latter number the numerator of a fraction, having for its denominator the former number. This fraction $\frac{7008}{9544}$ will express the probability of a person of 30 attaining to the age of 45.

The demonstration of this rule will be evident to those who understand the theory of probabilities.

PROB. II. *A young man, aged 20, borrows 1000l. to be paid with the interest when he attains to the age of 25; but in case he dies before that period, the debt to be cancelled: what sum ought the lender to receive at the proposed term of payment?*

It is here evident, that if it were certain that the young man would live to complete his twenty-fifth year, the sum to be paid would be the capital increased with 5 years interest, which we shall suppose to be at the rate of 5 per cent. or 1250*l.* But on account of the risk which the lender runs, by the chance of the borrower dying before the time of payment, the sum ought to be increased in the inverse ratio of the probability of his being alive. But this probability is expressed by the fraction $\frac{10259}{10909}$; and therefore the above sum must be multiplied by this fraction inverted, or by $\frac{10909}{10259}$, which will give nearly 1329*l.* that is say, about 79 more for the risk of losing the money, which certainly cannot be accounted usurious.

PROB. III. *A ſtate, or an individual, having occaſion to borrow a ſum of money on an annuity; what intereſt ought to be given for the different ages, the legal intereſt being 5 per cent?*

The vulgar, who are accuſtomed to burthenſome loans, entertain no doubt that an annuity, at the rate 10 per cent. for the age of 50, is a good bargain; and that this method of borrowing is advantageous to the ſtate; but they are egregiouſly miſtaken, for it appears by the tables of Pariceux, calculated from the foregoing data, that 10 per cent. ought not to be given before the age of 56; according to the ſame table no more than $6\frac{1}{5}$ per cent. ought to be given for the age of 20; $6\frac{1}{2}$ for the age of 25; $6\frac{4}{5}$ for that of 30; $7\frac{2}{3}$ for 40; $8\frac{4}{5}$ for 50; 10 at 56; $11\frac{1}{10}$ at 60; $16\frac{2}{5}$ at 70; $27\frac{2}{5}$ at 80; and $39\frac{1}{10}$ at 85.

It is therefore a great miſtake to imagine that, on account of the great number of perſons who ſink money in theſe loans, on annuities, made by governments, the latter are ſoon freed from paying a part of the annuities by the death of a part of the annuitants. The ſlow increaſe of annuities in tontines is a ſufficient proof of the falſity of this idea; beſides, the greatneſs of the number of the perſons is preciſely the cauſe, why the extinction of the annuities takes place more in conformity to the laws of probability already explained. A lucky chance, at the end of a few years, may free a perſon from the payment of an annuity, eſtabliſhed on the life of a man 30 years of age; but if this annuity were ſhared out on 300 different lives, the ages being nearly the ſame, it is certain that he would not be liberated from it before nearly 65 years; and after 32 or 33, nearly one half of the annuitants would be living. This Pariceux has clearly ſhewn by examining the liſts of the tontines.

MAGIC SQUARES.

A Magic ſquare is a ſeries of figures arranged in the cells of a ſquare, in ſuch a manner, that the figures in each band, whether vertical, horizontal or diagonal, form exactly the ſame ſum. They are divided into two kinds: odd and even.

Theſe ſquares have been called magic, becauſe the ancients aſcribed to them great virtues, and becauſe this arrangement of numbers formed the baſis and principle of ſeveral of their taliſmans.

One ſquare, containing unity, was according to them the ſymbol of the Deity, on account of his unity and immutability; for they obſerved that this ſquare, by its nature, was ſingle and immutable; the product of unity by itſelf being always unity.

A ſquare containing four diviſions or cells, was the ſymbol of imperfect matter, on account of the impoſſibility of arranging figures in it ſo as to form a magic ſquare.

The square with 9 divisions, was consecrated to Saturn; that with 16 to Jupiter; that with 25 to Mars; that with 36 to the Sun; that with 49 to Venus; that with 64 to Mercury; and that with 81, to the Moon.

Those who can find any relation between the arrangement of numbers and the planets, must be indeed not a little visionary; but such was the spirit of the mysterious philosophy of Iamblichus and Porphyry, and of all their disciples, who were slaves to the most stupid superstition, and to all the absurdities of judicial astrology.

We shall here confine ourselves to the mechanical method of forming a magical square, either even or odd.

Method of constructing an odd square.

1st. Place unity below the middle cell.

2d. Place the following numbers in the cells which descend diagonally from left to right.

3d. When you come to the last diagonal cell, go up to the highest cell, of the next following band.

4th. When the diagonal cell is filled up, carry the next figure to the most distant cell on the left of the lower band.

5th. In following the diagonal, if you meet with a cell already filled up, pass over that cell, and place the figure in the diagonal from right to left.

See the following figures, one of which represents a square of 9 divisions, and the other one of 25.

4	9	2
3	5	7
8	1	6

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Method of constructing an even square.

We shall apply this method to a square of 16 cells, which is filled up in the following manner:

1st. Place 1 in the cell A (fig. M.) of the vertical band on the left; then pass the two next, and place 4 in the upper cell of the perpendicular band, on the right.

2d. Omit 5, and place 6, 7, and the other figures, as seen in fig. M.

The remaining 8 divisions, which are left vacant, must be filled up after the manner of fig. N. Reckon 1 in the cell B without writing it down, and place 2 and 3 in the two next cells; then omit 4, and set down 5 in the first cell of the next band; omit 6 and 7, and write down 8, and so on. If you then fill up each of these squares from the other, you will have a square of 16 divisions. *See the figures.*

A geometrical square.

A

1			4
	6	7	
	10	11	
13			16

Fig. M.

	15	14	
12			9
8			5
	3	2	

B

Fig. N.

To arrange in a square, consisting of 9 cells, the 9 terms of a geometrical progression, in such a manner, that the product arising from the continued multiplication of the numbers in each band shall be always the same, and equal to the cube of the middle term.

Let the terms of the progression be 1 : 2 : 4 : 8 : 16 : 32 : 64 : 128 : 256. If you arrange these 9 terms in a square of 9 cells, in the manner as you did the 9 terms of the arithmetical progression of the natural numbers, 1, 2, 3, &c. you will find that the product of them, in every direction, amounts to 4096; which is exactly the cube of the middle term 16, as may be seen in the annexed figure.

8	256	2
4	16	64
128	1	32

To make the knight paſs over all the ſquares of the cheſs board, one after the other, without paſſing twice over the ſame.

As the reader may perhaps be unacquainted with the movement of the knight in the game of cheſs, we ſhall here deſcribe it. If the knight be placed in the ſquare A, he cannot be moved into any of the ſquares immediately around him, as thoſe marked 1, 2, 3, 4, 5, 6, 7, 8; nor into the ſquares 9, 10, 11, 12, which are directly above or below, or on one ſide; nor into the ſquares 13, 14, 15, 16, which are in the diagonals, but only into one of thoſe which, in the figure, are left vacant. See *fig.* B.

13		10		14
	1	2	3	
9	8	A	4	11
	7	6	5	
16		12		15

Fig. B.

Several celebrated men who amuſed themſelves with this problem, have given ſolutions of it; but the following is the ſimpleſt of them all, and the eaſieſt to be remembered.

34	49	22	11	36	39	24	1
21	10	35	50	23	12	37	40
48	33	64	57	38	25	2	13
9	20	51	54	63	60	41	26
32	47	58	61	56	53	14	3
19	8	55	52	59	62	27	42
46	31	6	17	44	29	4	15
7	18	45	30	5	16	43	28

The method conſiſts in filling up, as much as poſſible, the exterior bands which form as it were a border, without entering the third until there are no other means of paſſing from the ſquare at which you have arrived to one of the two firſt; a rule to which the knight is neceſſarily ſubjected, in the moſt evident manner, from his firſt ſtep to the 50th. When he arrives at the 50th, there is no other choice than 51 or 63; but the 51ſt ſquare, being nearer the border, ought to be preferred; and then his progreſs muſt neceſſarily be through 52, 53, 54, 55, 56, 57, 58, 59, 60, 61. When he arrives at the laſt, it is a matter of indifference whether he be made to paſs throught the three remaining ſquares, by directing his progreſs upwards or downwards; for in either caſe he will arrive at the laſt.

APPLICATION OF ANALYSIS

TO THE

SOLUTION OF VARIOUS PROBLEMS.

AS the object of this work is to unite instruction with amusement, we shall confine ourselves to such problems as are sufficiently easy to be solved by the application of those rules which we have explained in the introduction. (14)

PROB. I. *A lady lamenting that her age was triple that of her daughter; the latter consoled her by observing that in 15 years it would be only double: what was the age of each?*

Put a to denote the 15 years, and let x represent the age of the daughter; then by the conditions of the problem, the ages of the daughter and mother, which at present are x and $3x$, at the end of 15 years will be $x + a$ and $3x + a$; but as the age of the mother will then be double that of the daughter, we must multiply the age of the latter by 2, to have the following equation:

$$2x + 2a = 3x + a. \quad (14)$$

Then by transposition $2a - a = 3x - 2x$, (15)
And then by reduction $a = x$.

Consequently, the age of the daughter is 15, and that of the mother 45; which will answer all the conditions of the problem.

Prob. II. *A father, on his death-bed, gave orders in his will, that if his wife, who was then pregnant, brought forth a son, he should inherit $\frac{2}{3}$ of his property, and the mother the remainder; but if she brought forth a daughter, the latter should have only $\frac{1}{3}$, and the mother $\frac{2}{3}$. As the widow however was delivered of twins, a boy and a girl, what share ought each to have of the property left by the father?*

The only difficulty in this problem is to determine what would have been the will of the testator, had he foreseen that his wife would be delivered of twins. It has generally been explained in the following manner: As the testator desired that in case his wife brought forth a son, he should have two thirds of his property, and the mother one third, it hence follows that his intention was to give his son a sum double to that of the mother; and as he deſired, in the other caſe, that if ſhe brought forth a daughter, the mother ſhould have two thirds of his property, and the daughter one third, there is reaſon to conclude that he intended the ſhare of the mother to be double that of the daughter. Conſequently, to unite theſe two conditions, the heritage muſt be divided in ſuch a manner, that the ſon may have twice as much as the mother, and the mother twice as much as the daughter.

If a, therefore, be ſuppoſed to repreſent the father's property, and x the ſhare of the daughter, then $2x$ will expreſs that of the mother, and $4x$ that of the ſon. But, as all theſe ſhares together are equal to the father's property, we ſhall have the following equation:

$$x + 2x + 4x = a$$

By reduction $7x = a$

And by diviſion $x = \frac{a}{7}$. (16)

Hence, if we ſuppoſe the whole property to be 90000l. the daughter's ſhare will be 4285$\frac{5}{7}l$.; that of the mother 8571$\frac{3}{7}$, and that of the ſon 17142$\frac{6}{7}$.

Sometimes the following difficulty is propoſed in regard to this problem. In caſe the mother ſhould be brought to bed of two ſons and a daughter, in what manner muſt the property be divided.

In our opinion, no other anſwer can be given, than what would be given by the gentleman of the gown, viz. that in this caſe the will would be void;

for as no provision was made in it for a third child, its nullity would be established according to all the laws hitherto in existence. Because 1st. The law is precise. 2d. Because it is impossible to determine what would have been the dispositions of the testator if two sons had been born to him, or if he had foreseen that his wife would be delivered of two.

PROB. III. *A captain being asked how many soldiers he had in his company, replied: one half of them are in camp, one third in the trenches, one eighth in the hospital, and four in prison. Of how many men did his company consist?*

If the number of soldiers be expressed by x, and the four in prison by a, we shall have the following equation:

$$\frac{x}{2} + \frac{x}{3} + \frac{x}{8} + a = x.$$

Then by mult. $24x + 16x + 6x + 48a = 48x.$
By reduct. and transp. $48a = 48x - 46x = 2x.$
Consequently* $x = \frac{48a}{2} = 96.$

PROB. IV. *The head of a fish is 9 inches in length, its tail is as long as the head and half the body, and the body is as long as the head and the tail. What is the length of the fish?*

Let the head be expressed by a, the tail by x, and the body by y. By the conditions of the problem we ſhall then have the following two equations:

$$x = a + \frac{y}{2}$$

$$y = a + a + \frac{y}{2}$$

(16) By multiplication $\left.\begin{array}{l} 2x = 2a + y \\ 2y = 2a + 2a + y \end{array}\right.$

By tranſp. and reduction $y = 4a = 36.$

By this reſult the problem is ſolved; for the head being ſuppoſed equal to 9, the body denoted by $y = 36$, and the tail, being equal to the head and half the body, muſt neceſſarily be 27, which anſwers all the conditions of the problem.

PROB. V. *A perſon who had a leaſe of a houſe for 99 years, being aſked when it would expire, replied, that two-thirds of the time he had poſſeſſed it were exactly equal to four-fifths of the time unexpired. How many years of the leaſe were ſtill remaining?*

If we call the time elapſed x, and the 99 years a, the time unexpired will be $a - x$. Therefore, by the conditions of the problem,

$$\frac{2x}{3} = \frac{4a - 4x}{5}$$

By mult. $10x = 12a - 12x$

By transp. and reduct. $22x = 12a$

Consequently $x = \frac{12a}{22}$ or $\frac{6}{11}a = 54$.

Hence it appears, that as the time elapsed is 54 years, the period of the lease unexpired must necessarily be 45; and this solution agrees with the conditions of the problem.

PROB. VI. *It is proposed to divide the number 50 into two such parts, that the sum of three-fourths of the one, and five-sixths of the other may be equal to 40.*

Let $50 = a$, and $40 = b$; and if one of the parts of a be denoted by x, the other must necessarily be $a - x$.

By the conditions of the problem we shall then have the following equation:

$$\frac{3x}{4} + \frac{5a - 5x}{6} = b$$

By multiplication $18x + 20a - 20x = 24b$

By transp. and reduct. $20a - 24b = 20x - 18x = 2x$

By division $\frac{20a - 24b}{2} = x = 10a - 12b = 20$.

One of the parts of 50 then is 20, and the other 30; which answers the conditions of the problem; for 15, the three-fourths of 20, added to 25, the five-sixths of 30, is just equal to 40.

PROB. VII. *It is proposed to divide 100 into two such parts, that if a third of the one be taken from a fourth of the other, the remainder shall be 11.*

Let 100 = a, and 11 = b; also let one of the parts be expressed by x, and the other by $a - x$. Then $\frac{x}{3}$ will denote the third of the one part, and $\frac{a-x}{4}$ the fourth of the other; and by the conditions of the problem we shall have the following equation:

$$\frac{a-x}{4} - \frac{x}{3} = b$$

By multipli. $3a - 3x - 4x = 12b$

By transp. and reduct. $3a - 12b = 7x$

By division $\frac{3a - 12b}{7} = x = 24.$

The two parts of 100 then are 24 and 76; for if 8, the third of 24, be taken from 19, the fourth of 76, the remainder will be 11.

PROB. VIII. *Two persons sat down to play; one of whom had 72 guineas, and the other only 52; after a certain number of games they separated, the former carrying with him three times as many guineas as the other. How much did he win?*

Let a represent the 72 guineas of the former; b the 52 guineas of the latter, and x the loss of the second player.

The money of the first player when they give over play will therefore be $a + x$, and that of the other $b - x$; but as $a + x$, by the question, is three times as great as $b - x$, we shall have:

$$a + x = 3b - 3x$$

By transp. $$4x = 3b - a$$

By division $$x = \frac{3b - a}{4} = 21.$$

As it here appears that the loss of the second player was 21 guineas, leaving him only 31, the first must have carried off 93 guineas, which answers the conditions of the problem.

PROB. IX. *The minute hand of a clock being at 12, and the hour hand at 1, at what point between 1 and 2 will they both be in conjunction?*

If x represent the space, between the hours of 1 and 2 passed over by the hour hand before it is overtaken by the minute hand, and a the interval between 12 and 1; as the space passed over by the minute hand will be twelve times as great as that passed over by the hour hand, $a + x$ will be equal to $12x$; and we shall have the following equation:

$$a + x = 12x$$

By transp. and reduct. $\Big\}$ $a = 11x$

By division $\frac{a}{11} = x.$

From which we may conclude, that the minute hand will overtake the hour hand after the latter has passed over $\frac{1}{11}$ part of the space between the hours of 1 and 2.

Prob. x. *If two bodies move towards each other with unequal velocities, the ratio of which is known, as well as the distance between the bodies, to determine the point at which they will meet.*

Let the velocities be as 12 to 1, and let a represent the distance between the bodies, and x that part of it passed over by the body having the least velocity, when they meet.

The space then passed over by the body which has the greatest velocity will be $a - x$, and we shall have the following proportion:

$$12 : 1 :: a - x : x$$

By equation $12x = a - x$

By transp. and reduct. $\Big\}$ $13x = a$

By division $x = \frac{a}{13}.$

The solution of this problem is general; and consequently applicable to all cases where the distance of the bodies and the ratio of the velocities are known.

Prob. XI. *To divide 90 into two parts, which ſhall be to each other in the ſame ratio as 2 to 3.*

Let 90 be repreſented by a, the leaſt of the two parts by x, and the other by $a - x$. We ſhall then have the following proportion:

$$2 : 3 :: x : a - x$$

By equation $2a - 2x = 3x$

By tranſp. and reduct. $2a = 5x$

By diviſion $\frac{2a}{5} = x = 36.$

Conſequently the leaſt of the numbers will be 36, and the other 54; and indeed $36 + 54 = 90$, and $36 : 54 :: 2 : 3$.

Prob. XII. *Application of analyſis to the ſolution of the 11th problem of Divining Arithmetic, in which it is propoſed to tell the number of ſpots on all the bottom cards of ſeveral heaps, arranged on a table.*

It is here ſuppoſed that a complete pack of 52 cards is employed; and that as many cards are placed over the firſt of each heap as are neceſſary to make the ſum of the ſpots and cards together to amount to 12.

Let a reprefent 52, the whole number of cards, and b that of the remaining cards. The number of cards in all the heaps will then be $a - b$. If the number of fpots to be gueffed be expreffed by x, and the fum of all thefe fpots and the cards over them, as they are known, by c; we fhall have the following equation:

$$x + a - b = c$$

By tranfp. $x = c + b - a$.

That is to fay, if four heaps which are equivalent to a be deducted, x will be equal to the fum of the remaining cards, and the number of the fpots and cards which are in the other heaps. The truth of this operation may be eafily proved.

PROB. XIII. *What number is that, the $\frac{2}{3}$ of $\frac{3}{4}$ of which is equal to 1?*

Let x be the required number.

Then $\frac{2}{3}$ of $\frac{3x}{4} = 1$

But $\frac{2}{3}$ of $\frac{3x}{4} = \frac{6x}{12} = \frac{x}{2}$.

Confequently $\frac{x}{2} = 1$, or $x = 2$.

Proof: $\frac{3}{4}$ of 2 are $\frac{3}{2}$; and $\frac{2}{3}$ of $\frac{3}{2} = \frac{6}{6} = 1$.

PROB. XIV. *What number is that, $\frac{3}{4}$ of $\frac{2}{3}$ of which $+ \frac{1}{2}$ of $\frac{1}{4}$ of it, is equal to 11?*

Let x, as before, be the required number.

Then $\frac{3}{4}$ of $\frac{2}{3}$ of x are $\frac{x}{2}$

And $\frac{1}{2}$ of $\frac{5}{6}$ of x is $\frac{5x}{12}$

But by the supposition $\frac{x}{2} + \frac{5x}{12} = 11$

Therefore $11x = 11 \times 12$, and $x = 12$.

Proof: $\frac{2}{3}$ of 12 are 8, and $\frac{3}{4}$ of 8 are 6 $= \frac{3}{4}$ of $\frac{2}{3}$ of 12; $\frac{5}{6}$ of 12 are 10, and the half of 10 is 5 $= \frac{1}{2}$ of $\frac{5}{6}$ of 12: But $5 + 6 = 11$. Therefore, &c.

PROB. XV. *What number is that, $\frac{2}{3}$ of $\frac{4}{5}$ of which — $\frac{1}{2}$ of $\frac{3}{4}$ of it are equal to 19?*

First, $\frac{1}{2}$ of $\frac{3}{4}$ of x is $\frac{3x}{8}$

And $\frac{2}{3}$ of $\frac{4}{5}$ of x are $\frac{8x}{15}$;

Then $\frac{8x}{15} - \frac{3x}{8} = \frac{64x}{120} - \frac{45x}{120} = \frac{19x}{120}$.

But $\frac{19x}{120}$ equal 19 by the problem. Therefore, $19x = 19 \times 120$; and $x = 120$.

Proof: $\frac{2}{3}$ of $\frac{4}{5}$ of 120 are 64, and $\frac{1}{2}$ of $\frac{3}{4}$ of 120 is 45; but $64 - 45 = 19$. Therefore, &c.

PROB. XVI. *What number is that, of which $\frac{2}{3}$ of $\frac{3}{4}$ multiplied by $\frac{1}{2}$ of $\frac{1}{6}$ of it will be equal to 6?*

$\frac{2}{3}$ of $\frac{3}{4}$ of x are $\frac{x}{2}$; and $\frac{1}{2}$ of $\frac{1}{6}$ of x is $\frac{x}{12}$, and $\frac{x}{2} \times \frac{x}{12} = \frac{x^2}{24}$: then by the conditions of the problem, $\frac{x^2}{24} = 6$. Therefore $x^2 = 144$; and consequently $x = 12$.

Proof: $\frac{2}{3}$ of $\frac{3}{4}$ of 12 are 6; and $\frac{1}{2}$ of $\frac{1}{6}$ of 12 is 1; but $6 \times 1 = 6$. Therefore, &c.

PROB. XVII. *What number is that, of which $\frac{1}{2} + \frac{3}{4}$ are equal to 1?*

Let x be the number required.

Then $\frac{x}{2} + \frac{3x}{4} = 1$, or $\frac{5x}{4} = 1$

Therefore $5x = 4$

Consequently $x = \frac{4}{5}$.

Proof: $\frac{1}{2}$ of $\frac{4}{5} = \frac{2}{5}$; and $\frac{3}{4}$ of $\frac{4}{5} = \frac{3}{5}$; but $\frac{2}{5} + \frac{3}{5}$ or $\frac{5}{5} = 1$. Therefore, &c.

PROB. XVIII. *What number is that, the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of which make 12?*

Let x be the required number.

Then $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 12$

or $12x + 8x + 6x = 24 \times 12$

Therefore $26x = 288$

And $x = \frac{288}{26} = 11\frac{1}{13}$.

Proof: $\frac{1}{2}$ of $11\frac{1}{13}$ is $5\frac{7}{13}$; $\frac{1}{3}$ of $11\frac{1}{13}$ is $3\frac{9}{13}$; and $\frac{1}{4}$ of $11\frac{1}{13}$ is $2\frac{10}{13}$; but $5\frac{7}{13} + 3\frac{9}{13} + 2\frac{10}{13} = 12$.

I

PROB. XIX. *The triple, the half, and the fourth of a certain number, are equal to 104: What is the number?*

Let x be the number required. We ſhall then have, by the conditions of the problem:

$$3x + \frac{x}{2} + \frac{x}{4} = 104$$

Therefore $30x = 104 \times 8 = 832$

Conſequently $x = \frac{832}{30} = 27\frac{11}{15}$.

Proof: $27\frac{11}{15} \times 3 = 83\frac{3}{15}$

$\frac{1}{2}$ of $27\frac{11}{15} = 13\frac{13}{15}$

$\frac{1}{4}$ of $27\frac{11}{15} = 6\frac{14}{15}$

The ſum 104

PROB. XX. *If $\frac{3}{4}$ and $\frac{1}{6}$ of the hull of a ſhip be immersed in the sea, and only 4 feet of it above the surface of the water: What is the depth of the veſſel?*

Let x be the depth of the veſſel.

Then $\frac{3x}{4} + \frac{x}{6} + 4 = x$

or $18x + 4x + 96 = 24x$

Therefore $2x = 96$

And $x = 48$ feet, the depth of the veſſel.

Proof: $\frac{3}{4}$ of $48 = 36$

$\frac{1}{6}$ of $48 = 8$

44

Feet above water 4

48

PROB. XXI. *A banker at his death, being deſirous to reward 10 of his clerks, gave orders in his will,* *that 5500 guineas ſhould be divided among them, in ſuch a manner, that the firſt 5 ſhould have each an equal ſhare of the whole legacy; that the next 3 ſhould have ſhared among them one-half of what was bequeathed to the firſt 5; and that the 2 laſt ſhould have divided between them one-third of that ſum: What was the ſhare of each?*

Let x be the ſhare of each of the firſt five clerks, and $a =$ the 5500 guineas.

Then, by the conditions of the problem, the ſhare of the firſt five will be $5x$; that of the next three $\frac{5}{2}x$; and that of the two laſt $\frac{5}{3}x$.

But as theſe three quantities are equal to a, or the whole, we have the following equation:

$$5x + \frac{5}{2}x + \frac{5}{3}x = a.$$

By multip. and reduct. $55x = 6a$

By diviſion $x = \frac{6a}{55} = 600$ guineas.

Each of the firſt five then had......600

Each of the next three............500

And each of the two laſt..........500

Proof:
$$\begin{cases} 5 \times 600 = 3000 \\ 3 \times 500 = 1500 \\ 2 \times 500 = 1000 \end{cases}$$
$$5500$$

The application which we have here made, of analyſis to the ſolution of a few problems, evidently ſhews that this method, by its preciſion, brevity, and extent, is far ſuperior to arithmetic. The latter confines our attention to determinate quantities, and, if I may uſe the expreſſion, enchains it by the ſlowneſs of its progreſs ; while the other, more rapid, enables us to paſs over the intermediate operations, and to direct our attention to the real point of difficulty.

The chief advantages, therefore, derived from this ſcience are, that it facilitates the diſcovery and comprehenſion of mathematical truths, and that it ſupplies us with eaſy methods, and general rules, for reſolving all problems that may be propoſed reſpecting quantities.

When we have obtained a reſult by the rules of arithmetic, there is nothing indeed that exhibits to the mind the chain of operations which conducted to it. When, after a few arithmetical operations, we have obtained 12 for reſult, we ſee nothing in 12 which can indicate whether this number has ariſen from the multiplication of 3 by 4, of 2 by 6, or by the addition of 5 to 7, or of 2 to 10 ; or, in general, from the combination of any other operations. Arithmetic gives rules for finding certain reſults ; but theſe reſults of themſelves can furniſh no rules. Algebra, however, or that mode of calculation which employs indeterminate characters, preſerves, as we may ſay, the traces of all the intermediate operations, which conduct to the laſt reſult.

PALINGENESY.

PALINGENESY is a chemical operation, by means of which a plant or an animal, as some pretend, can be revived from its ashes. This, if true, would no doubt be one of the noblest secrets of chemistry and philosophy. If some authors are to be believed, several learned men of the 17th century were in possession of it; but at present as this pretended secret, in consequence of the great progress made in chemistry, is considered as a mere chimera, we shall here confine ourselves to examining the foundation of those principles, which have induced some respectable authors, such as the Abbé Vallemont, and others, to believe in the possibility of this process.

According to the good Abbé, nothing is simpler and easier to be explained. We are indeed told, says he, by Father Kircher, that the seminal virtue of each mixture, is contained in its salts; and these salts, unalterable by their nature, when put in motion by heat, rise in the vessel through the liquor in which they are diffused. Being then at liberty to arrange themselves at pleasure, they resume their primitive disposition, and place themselves in that order in which they would be placed by the effect of vegetation, or the same as they occupied before the body to which they belonged had been decomposed by the fire: in a word, they form a plant, or the phantom of a plant, which has a perfect resemblance to the one destroyed.

This reasoning is worthy of an author who could believe, that he who robs another of his money, can exhale corpuscules different from those exhaled by a man who carries his own, and thereby make the divining rod turn towards the places where he has passed, or remained some time. Does it not shew great weakness to believe, that the mere immorality of an action can produce physical effects? It would indeed be offering an insult to our readers, to attempt to shew the folly and absurdity of the above reasoning of the good Abbé, and of Father Kircher. Let us therefore only examine the facts which he relates.

An English chemist, named Coxe, asserts, that having extracted and dissolved the essential salts of fern, and then filtered the liquor, he observed, after leaving it at rest for five or six weeks, a vegetation of small ferns, adhering to the bottom of the vessel.

The same chemist having mixed northern pot-ash with an equal quantity of ſal ammoniac, ſaw, ſome time after, a ſmall foreſt of pines and other trees, with which he was not acquainted, riſing from the bottom of the veſſel.

The following fact the author thinks concluſive. The celebrated Boyle, though not very favourable to palingeneſy, relates, that having diſſolved in water ſome verdegris; which, as is well known, is produced by combining copper with the acid of vinegar, and having cauſed this water to congeal by means of artificial cold, he obſerved at the ſurface of the ice ſmall figures, which had an exact reſemblance to vines.

Notwithſtanding theſe facts, and ſeveral others mentioned by the Abbé Vallemont, if the partiſans of palingeneſy can produce none more concluſive, it muſt be confeſſed that they ſupport their pretenſions by very weak proofs. Every true chemiſt ſees in theſe phenomena, nothing but a ſimple ramified cryſtallization, which may be produced by means of different well-known compoſitions: the moſt beautiful of theſe cryſtallizations, called improperly *vegetations*, are produced by the combination of bodies from the animal kingdom.

The laſt experiment related by Boyle might occaſion more embarraſſment; but as it is the only one, of a great many, made with the eſſential ſalts of a variety of plants, that ſucceeded, there can be no doubt that the figures he ſaw were the mere effect of chance; for how many other philoſophers, who made the ſame attempt, ſaw nothing but what is uſually exhibited by the ſurface of frozen water, which ſometimes forms ramifications exceedingly complex?

The partiſans of palingeneſy quote, however, other authorities, to which they attach great importance. We are told by Sir Kenelm Digby, on the report of Querceran, phyſician to Henry IV. of France, that a Pole ſhewed twelve glaſs veſſels, hermetically ſealed, each containing the ſalts of different plants; that at firſt theſe ſalts had the appearance of aſhes, but that when expoſed to a gentle and moderate heat, the figure of the plant, ſuch as a roſe for example, if the veſſel contained the aſhes of a roſe, was obſerved gradually to riſe up, and that as the veſſel cooled the whole diſappeared. Sir Kenelm adds, that Father Kircher had aſſured him, he had performed the ſame experiment, and that he communicated to him the ſecret; but it had never ſucceeded. The ſtory of this Pole is related by various other authors, ſuch as Guide la Broſſe, in his book on the Nature of Plants.

In the laſt place, we are told by Kircher himſelf, in his *Ars Magnetica*, that he had a long-necked phial hermetically ſealed, containing the aſhes of a plant, which he could revive when he thought proper, by means of heat, and that he ſhewed this wonderful phenomenon to Christina queen of Sweden, who was highly delighted with it; but that, having left this valuable curiosity one cold day in his window, it was entirely destroyed by the frost. Father Schott also asserts, that he saw this chemical wonder, which according to him was a rose revived from its ashes; and he adds, that a certain prince having requested Kircher to make him one of the same kind, he chose rather to give up his own, than to repeat the operation.

The process, indeed, as taught by Kircher, is so complex and tedious, that it would require no small patience to follow it. Father Schott relates it at full length, in his work intituled *Jocoseria Naturæ et Artis*; and he calls it the imperial secret, because the emperor Ferdinand purchased it from a chemist, and gave it to Kircher. This emperor was exceedingly fortunate, for it was to him that the philosopher who had the secret of the philosopher's stone addressed himself; and gave a proof of his art, by transmuting, as is said, in his presence, three pounds of mercury into two pounds and a half of gold.

We must, however, be satisfied with pointing out the places where the curious may find this singular process; for, besides the length of the description, nothing seems less calculated to succeed. Digby, therefore, and many others, who followed this method, did not ſucceed; and there is reaſon to believe, that their zeal for palingeneſy would induce them to omit nothing that was likely to inſure a favourable reſult.

Dobrezenſky of Negropont has alſo given a proceſs for the reſurrection of plants, which ſeems to have been attended with no better ſucceſs. We are at leaſt told by Father Schott, that the attempts of Father Conrad proved ineffectual; and he therefore ſuppoſes that Dobrezenſky did not reveal all the circumſtances of the proceſs, but kept the moſt important to himſelf.

What then can be ſaid in oppoſition to all theſe authorities?—We ſhall only obſerve, that the Poliſh phyſician was a quack; and ſhall deſcribe a method of producing a falſe palingeneſy, which if performed with art, and in a proper place, may impoſe on credulous perſons. To be convinced that Dobrezenſky of Negropont was a mere impoſtor, we need only read the *Technica Curioſa*, or the *Joco-ſeria Naturæ et Artis* of Father Schott; for he had the impudence to pretend that he could pull out the eye of any animal, and in the courſe of a few hours reſtore it by means of a liquor which he no doubt ſold as a remedy for ſore eyes. He even made trial of it on a cock. A perſon who could aſſert ſuch an impudent falſehood in regard to one fact, would do the ſame in regard to another.

The authority of Father Schott will certainly be-of very little weight with those who have read his works.

In regard to Kircher, however celebrated he may have been in the arts and sciences, we shall not hesitate to reject his testimony; for he was a man of a vivid imagination, passionately fond of every thing singular and extraordinary, and who had a strong propensity to believe in the marvellous. What can be expected from a man of such a character! He often thinks he sees what he does not see; and he does not deceive others, because he is first deceived himself.

Some go still farther, and assert that an animal may be revived from its ashes. Father Schott, in his *Physica Curiosa*, even gives the figure of a sparrow thus revived in a bottle. Caffarel, in his *Unheard of Curiosities*, believes in this fact, and considers it as a proof of the possibility of the general resurrection of bodies. This pretended revival however is a chimera, still more ridiculous than the former; and which at present it would be ridiculous to attempt seriously to refute.

In a word, what reasonable man can with Kircher believe, that if the ashes of a plant be scattered on the ground, plants of the like kind will spring up from them, as he says he frequently experienced? Who can admit as truth, that if crabs be burnt and then distilled, according to a process given by Digby, there will be produced in the liquor ſmall crabs of the ſize of a grain of millet, which muſt be nouriſhed with ox's blood, and then left to themſelves in ſome ſtream? Yet we are told by the Engliſh knight, that this he himſelf experienced. It is therefore impoſſible to clear him from the charge of impoſture, unleſs we ſuppoſe that, by ſome means or other he was led into an error. However this may be, it is certain that Digby, with great zeal, and a conſiderable ſhare of knowledge, had a ſtrong propenſity to all the viſions of the occult and cabaliſtic ſciences.

An illuſory kind of Palingeneſy.

By the following deception, credulous people may be eaſily impoſed upon, and induced to believe in the reality of Palingeneſy.

Provide a double glaſs jar of a moderate ſize, that is to ſay, a veſſel formed of two jars placed one within the other, in ſuch a manner, that an interval of only a line in diameter may be left between them. The veſſel may be covered with an opake top or lid, ſo diſpoſed, that by turning it in different directions, the inner jar may be raiſed from, or brought nearer to the bottom of the exterior one. In the interior jar, on a baſe repreſenting a heap of aſhes, place the ſtem of an artificial roſe. Into the lower part of the interval between the two jars introduce a certain quantity of ashes, or some solid substance of a similar appearance, and let the remainder be filled with a composition made of one part of white wax, twelve parts of hog's lard, and one or two of clarified linseed oil. This oily compound, when cold, will entirely conceal the inside of the jar; but when brought near the fire, if done with dexterity, it will dissolve, and by shaking the lid, under a pretence of hastening the operation, the compound may be made to fall down into the bottom of the exterior jar. The rose in the interior one will then be seen, and the credulous spectators, who must not be suffered to approach too near, will be surprised and astonished. When you wish to make the rose disappear, remove the jar from the fire, and by a new slight of hand make the dissolved semi-transparent wax flow back into the interval between the jars. By accompanying this manœuvre with proper words, the gaping spectators will be more easily deceived; and will retire firmly persuaded, that they have seen one of the most curious phenomena that can be exhibited by the united efforts of chemistry and philosophy.

Palingenesy; or the art of reviving the dead, and making the image of a deceased person appear in a glass jar.

A juggler at Venice took a glass jar, and having poured water into it, told the company, of whom

I formed one, that he would make the image of any deceased person we might name appear in it. One of the company desired that he might see his grandfather, and actually believed that he distinguished his image in the jar.

To explain the cause of this phenomenon, we must observe, that concave mirrors differ from plane ones by producing different effects.

The image of an object placed before a vertical plane mirror appears behind it, at a distance equal to that which is between it and the surface of the mirror, in the same position and of the same size; but an object placed before a concave mirror, beyond the focus of its concavity, appears inverted before the mirror, and smaller than it really is. By a natural consequence therefore, an inverted object, by means of such a mirror, must appear in its proper position.

This being the case, if an inverted object be concealed, at a proper distance from the bottom of a box, furnished with a concave mirror, the object will appear in its natural position, towards a small aperture made in the box near the focus of reflection.

Now if several inverted figures be arranged round a circle, supported in an horizontal manner on a pivot, like the card of a mariner's compass, by the help of a magnet or thread the circle may be turned round at pleasure, so as to present to the mirror any of the figures, which will of course appear in the jar.

Before the required figure is exhibited, some questions are generally asked the spectator, respecting the age, the character and physiognomy of the person who is to appear; and by these means the conjuror has it in his power to produce a figure corresponding with the description given. If the spectator complains that the figure has no great resemblance to the deceased, (which will seldom happen, as the power of imagination concurs to deceive him), the conjuror tells him that he does not pretend to exhibit the person in the state in which he was, while in perfect health, but pale and disfigured, such as he appeared a few moments before his death.

To prove that the jar contains a reviving power, artificial flowers, placed in an inverted order, may be used instead of the above figures; if one of them be burnt, and the ashes thrown into the jar, the image of it may be made to appear in the inside as before.

THE AMIANTHUS.

THE Amianthus, or Asbestos, is a kind of stone of a greyish colour, which may be divided into hard coriaceous threads or filaments. These filaments are disposed sometimes in a parallel order, sometimes in bundles, and sometimes are mixed in an irregular manner. They are united together by a calcareous matter; but if this matter be softened in water, they may be easily separated.

Most kinds of this stone oppose so effectual a resistance to the action of fire, that it only whitens them, by dissipating those foreign bodies which obscure their splendour, and render them impure. The filaments of this mineral are sometimes so flexible and soft, that they may be spun and wove.

Purses, girdles, garters, and other small articles of spun amianthus, are preserved in the cabinets of the curious; and we learn from history, that formerly the bodies of great men were burnt in cloth made of this substance, in order to preserve their ashes pure and unmixed with those of the funeral pile. Cloth of this kind, by being thus exposed to the fire, became whiter and more beautiful, without experiencing any other alteration, than a small decrease in its weight.

The process employed to render amianthus fit for being spun, consists in first steeping it in warm water, and rubbing it with the hands, to free it from all foreign matters; it must then be carded, and immersed in oil, to render it pliable; after which it is spun along with cotton or wool. When the cloth is wove, it is thrown into the fire, by which means the wool or other materials employed to render it easier to be spun, are consumed, and nothing remains but the amianthus.

This ſubſtance might be converted alſo into a kind of incombuſtible paper, exceedingly uſeful for public records, or private deeds, the preſervation of which is often of great importance. Such records would be in no danger of being burnt; but it would be neceſſary to diſcover ſome kind of ink capable of reſiſting the action of fire alſo.

To make paper of amianthus, it muſt be pounded, in order to bring it to the ſtate of lint or cotton, and by employing a ſieve the ſtoney part it contains will paſs through, and leave nothing but the amianthus: it muſt then be formed into a paſte, and ſubjected to the ſame operations as thoſe uſed for making common paper. This paper however has hitherto been attended with this inconvenience, that it is rather brittle; but ſome method may perhaps be found to bring it to greater perfection.

A kind of paper, which burns with difficulty, and which, on that account, is exceedingly uſeful for wrapping up valuable articles, or ſuch as are liable to catch fire by the leaſt ſpark, may be prepared by a very ſimple proceſs: nothing more being neceſſary, but to draw common paper two or three times through a boiling ſolution, of one part of alum, in three parts of water.

ACOUSTICS AND MUSIC.

THE ancients ſeem to have conſidered ſounds under no other point of view than that of muſic; that is to ſay, as affecting the ear in an agreeable manner: it is even very doubtful whether they were acquainted with any thing more than melody, and whether they had any art ſimilar to that which we call compoſition. The moderns however, by attending to the philoſophy of ſounds, have made many diſcoveries in this department, ſo much neglected by the ancients; and hence has ariſen a new ſcience, diſtinguiſhed by the name of acouſtics. Acouſtics have for their object the nature of ſounds conſidered, in general, both in a mathematical and philoſophical view. This ſcience therefore comprehends muſic, which conſiders the ratios of ſounds, ſo far as they are agreeable to the ear, either by their ſucceſſion, which conſtitutes melody; or by their ſimultaneiety, which forms harmony. We ſhall here give a brief account of every thing moſt curious and intereſting in regard to this ſcience.

Definition of ſound; how diffuſed and tranſmitted to our organs of hearing; experiments on this ſubject; different ways of producing ſound.

Sound is nothing elſe but the vibration of the particles of the air, occaſioned either by ſome ſudden agitation of a certain maſs of the atmoſphere, violently compreſſed or expanded, or by the communication of the vibration of the inſenſible parts of a hard and elaſtic body.

Theſe are the two beſt known ways of producing ſound. The exploſion of a piſtol, or of any other kind of fire arms, produces a report or ſound, becauſe the air or elaſtic fluid, contained in the gunpowder, being ſuddenly dilated, compreſſes the external air with great violence: the latter in conſequence of its elaſticity, re-acts on the ſurrounding atmosphere, and produces in its moleculæ an oſcillatory motion, which occaſions the ſound, and which extends to a greater or leſs diſtance, according to the intenſity of the cauſe that gave riſe to it.

The other method of producing ſound, is to excite in an elaſtic body, vibrations ſufficiently rapid to occaſion, in the ſurrounding parts of the air, a ſimilar motion. Thus, an extended ſtring, when ſtruck, emits a ſound; and its oſcillations, that is to ſay its motion backward and forward, may be diſtinctly ſeen. The elaſtic parts of the air ſtruck by the ſtring, during the time it is vibrating, are themſelves put into a ſtate of vibration, and communicate this motion to the neighbouring ones. Such alſo is the mechaniſm by which a bell produces its ſound: when ſtruck, its vibrations are ſenſible to the hand which touches it.

That air is the vehicle of ſound, may be proved by the following experiment: if a bell be ſuſpended in the receiver of an air-pump, the ſound of it decreaſes in proportion as the air is exhauſted, and at laſt becomes totally inſenſible when a complete vacuum has been formed.

Sound always ceaſes when the vibrations of the ſonorous body ceaſe, or become too weak. This may be proved alſo by an experiment; for when the vibrations of a ſonorous body are damped by any ſoft body, the ſound ſeems ſuddenly to ceaſe: in a piano-forte therefore the quills are furniſhed with bits of cloth, that by touching the ſtrings when they fall down, they may damp their vibrations. On the other hand, when the ſonorous body is, by its nature, capable of continuing its vibrations for a conſiderable time, as is the caſe with a large bell, the ſound may be heard for a long time after.

Of the velocity of ſound; experiments for determining it; method of meaſuring diſtances by it.

Light is tranſmitted from one place to another with inconceivable velocity; but this is not the caſe with found: the velocity of found is very moderate, and may be meafured in the following manner.

Let a cannon be placed at the diftance of feveral thoufand yards, and let an obferver, with a pendulum that vibrates feconds, or rather half feconds, put the pendulum in motion, as foon as he fees the flafh, and then count the number of feconds or half feconds which elapfe between that period and the moment when he hears the explofion. It is evident, that if the moment when the flafh is feen be confidered as the fignal of the explofion, nothing will be neceffary, to obtain the number of yards which the found has paffed over in a fecond or half fecond, but to divide the number of the yards, between the place of obfervation and the cannon, by the number of the feconds or half feconds which have been counted.

Now the moment when the flafh is perceived may be confidered as the real moment of the explofion; for fo great is the velocity of light, that it employs fcarcely a fecond to traverfe 70000 leagues.

By this method it has been found, that found moves at the rate of about 1142 feet in a fecond.

This method may be employed to determine the diftance of fhips at fea, or in a harbour, when they fire guns, provided the flafh can be feen, and the explofion heard. During a ftorm alfo the diftance of a thunder-cloud may be determined in the ſame manner. But as a pendulum is not always to be obtained, its place may be ſupplied by obſerving the beats of the pulſe; for when in its uſual ſtate each interval, between the pulſations, is almoſt equal to a ſecond.

How ſounds may be propagated in every direction, without confuſion.

This is a very ſingular phenomenon in the propagation of ſounds; for if ſeveral perſons ſpeak at the ſame time, or play on inſtruments, their different ſounds are heard ſimultaneouſly, or all together, either by one perſon, or by ſeveral perſons, without being confounded in paſſing through the ſame place in different directions. Let us endeavour to account for this phenomenon.

The moleculæ of the air contiguous to the ſonorous body receive from it an oſcillatory and vibratory motion, which, in conſequence of their elaſticity, is ſucceſſively tranſmitted to a certain diſtance. As the ſonorous body is the centre from which the motion is communicated in every direction, the ſound muſt neceſſarily become weaker in proportion as the maſs of air, which receives it, becomes greater. The different ſounds, of whatever nature, muſt be heard, becauſe they are tranſmitted to the organ of hearing by analogous moleculæ of the air, in the same manner as when a certain tone is emitted, in an apartment, it cannot be repeated but by thoſe ſtrings of the inſtrument which are in uniſon with it. Sounds of greater intenſity cannot be propagated with more velocity, though the vibrations of the aërial moleculæ which tranſmit them be ſtronger, becauſe they are always iſochronous, like thoſe of pendulums more or leſs removed from the vertical line, or ſtrings more or leſs bent.

Of echoes; how produced; account of the moſt remarkable echoes, and of ſome phenomena reſpecting them.

Echoes are well known; but however common this phenomenon may be, it muſt be allowed that the manner in which it is produced, is involved in conſiderable obſcurity; and that the explanation given of it does not ſufficiently account for all the circumſtances attending it.

All philoſophers almoſt have aſcribed the formation of echoes to a reflexion of ſound, ſimilar to that experienced by light, when it falls on a poliſhed body; but, as D'Alembert obſerves, this explanation is falſe; if it were not, a poliſhed ſurface would be neceſſary for the production of an echo; but it is well known that this is not the caſe. Echoes indeed are frequently heard oppoſite to old walls, which are far from being poliſhed; near ſhapeleſs maſſes of rock, and in the neighbourhood of foreſts, and even of clouds. This reflexion of ſound therefore is not of the ſame nature as that of light.

It is evident however, that the formation of an echo can be aſcribed only to the repercuſſion of ſound; for echoes are never heard, but when ſound is intercepted and made to rebound by one or more obſtacles.

Sound, as already ſaid, is propagated in every direction by the vibration of the particles of the air; but if any column of air reſts againſt ſome obſtacle that prevents the direct movement of the elaſtic globules, which ſerve as the vehicle of ſound, it muſt rebound in a contrary direction, and ſtriking the ear, if it meets with one in the line of repercuſſion, convey to it a repetition of the ſame ſound, provided the original ſound does not affect that organ at the ſame inſtant.

But we are taught by experience that the ear does not diſtinguiſh the ſucceſſion of two ſounds, unleſs there be between them the interval of at leaſt one twelfth of a ſecond; for during the moſt rapid movement of inſtrumental muſic, each meaſure of which cannot be eſtimated at leſs than a ſecond, twelve notes are the utmoſt that can be comprehended in a meaſure, to render the ſucceſſion of the ſounds diſtinguiſhable; conſequently the obſtacle, which reflects the ſound, muſt be at such a distance, that the reverberated sound shall not succeed the direct sound, till after one twelfth of a second; and as sound moves at the rate of about 1142 feet in a second, and consequently about 95 feet in the twelfth of a second, it thence follows that, to render the reverberated sound distinguishable from the direct sound, the obstacle must be at the distance of no more than about 48 feet.

There are single and compound echoes. In the former, only one repetition of the sound is heard; in the latter, there are 2, 3, 4, 5, &c. repetitions. We are even told of echoes that can repeat the same word 40 or 50 times.

Single echoes are those where there is only one obstacle; but double, triple, or quadruple echoes, give us reason to suppose several obstacles disposed in such a manner, that the different reflected sounds strike the ear at times sensibly different.

There are some echoes that repeat several words in succession; but this is not astonishing, and must always be the case when a person is at such a distance from the echo, that there is sufficient time to pronounce several words before the repetition of the first has reached the ear.

There are certain echoes which have been much celebrated on account of their singularity, or of the number of times that they repeat the same word.

Miſſon, in his deſcription of Italy, ſpeaks of an echo, in the vineyard of Simenetta, which repeated the ſame word 40 times.

At Woodſtock, in Oxfordſhire, there is an echo which repeats the ſame ſound 50 times.

The deſcription of an echo ſtill more ſingular, near Roſneath, ſome miles diſtant from Glaſgow, may be found in the Philoſophical Tranſactions, for the year 1698. If a perſon, placed at the proper diſtance, plays 8 or 10 notes of an air with a trumpet, the echo faithfully repeats them, but a third lower; after a ſhort ſilence, another repetition is heard, in a tone ſtill lower; and another ſhort ſilence is followed by a third repetition, in a tone a third lower.

A ſimilar phenomenon obſerved in ſome places is, that if a perſon ſtands in a certain poſition, and pronounces a few words with a low voice, they are heard only by another perſon ſtanding in another determinate place: this ariſes from the elliptic form of arches, which have the property of collecting in one of their foci the rays that proceed diverging from the other.

The following phenomenon depends on the ſame theory.

To conſtruct two figures, to be placed at the two ends of a hall, one of which ſhall repeat to the ear of a perſon what has been whiſpered into the ear of *the other figure, without being heard by any other person in the hall.*

Provide two heads or busts, made of pasteboard, resting on pedestals, and place them in a hall at such a distance from each other as you may think proper. Then convey a tube of tin-plate, an inch in diameter, from the ear of one of the figures, through the pedestal on which it rests, and below the flooring, till it reach the mouth of the other figure, passing through its pedestal in the same manner, as that of the former: this tube must be a little wider, at each of its extremities, somewhat in the form of a funnel.

When it is necessary to bend this tube, care must be taken to cover the interior angles with a piece of tin-plate inclined at an angle of 45 degrees, that the voice may be directly reflected from one part of the tube to the other, and that the sound may be conveyed distinctly to the ear.

This construction will produce the following effect. If a person whispers into the ear of one of these figures, the words he pronounces will be distinctly heard by a second person who applies his ear to the mouth of the other figure.

The secret of the magic mirror, as it is called, depends on the same theory. The construction of this mirror is as follows:

Fix, in a vertical poſition, a concave mirror, two feet in diameter, and of ſuch a degree of curvature, that the focus of the rays which fall upon it, in a parallel direction, may be at the diſtance of twelve or fifteen inches from the reflecting ſurface. At this diſtance place a ſmall figure, but in ſuch a manner, that its head may be exactly in the focus.

This mirror muſt be placed at the diſtance of 8 or 10 feet from a wall oppoſite to it, and parallel to its ſurface: the wall muſt have in i an aperture, equal to the ſurface of the mirror, concealed by a very fine curtain, that the ſound may eaſily paſs throught it. Provide alſo a ſecond mirror of the ſame form, with a ſimilar figure, and place it behind the wall at the diſtance of two or three feet from it, and oppoſite to the former, with the figure in its focus. It may be readily conceived, that when a perſon only whiſpers into the ear of the ſmall figure behind the wall, a perſon ſtanding near that placed in the focus of the oppoſite mirror, will hear very diſtinctly the words whiſpered into the ear of the former. In this manner, the perſon who aſks a queſtion, ſtanding near the firſt figure, hears the anſwer which is whiſpered into the ear of the other behind the wall.

In order to conceal entirely the apparatus, which produces this effect, and to render it much more extraordinary, the pretended concave magic mirror may be covered with a piece of gauze, which will not prevent the tranfmiffion of the founds from the one focus to the other.

The Memoirs of the Academy of Sciences, at Paris, for the year 1692, fpeak of a very remarkable echo in the court of a gentleman's feat, called le Genetay, in the neighbourhood of Rouen. It is attended with this fingular phenomenon, that a perfon who fings or fpeaks in a low tone does not hear the repetition of the echo, but only his own voice; while, on the other hand, thofe who liften hear only the repetition of the echo, but with furprifing variations; for the echo feems fometimes to approach and fometimes to recede, and at length ceafes when the perfon who fpeaks removes to fome diftance in a certain direction. Sometimes only one voice is heard, fometimes feveral, and fometimes one is heard on the right and another on the left. An explanation of all thefe phenomena, deduced from the femi-circular form of the court, may be feen in the above collection.

Experiments refpecting the vibrations of mufical ftrings, which form the bafis of the theory of mufic.

If a ftring of metal or cat-gut, fuch as is ufed for mufical inftruments, made faft at one of its extremities, be extended in a horizontal direction over a fixed bridge, and a weight be suspended from the other extremity, so as to stretch it; this string, when struck, will emit a sound produced by reciprocal vibrations which are sensible to the sight.

If the part of the string made to vibrate be shortened, and reduced to one half of its length, any person who has a musical ear will observe, that the new sound is the octave of the former; that is, to say, twice as sharp.

If the vibrating part of the string be reduced to two thirds of the original length, the sound it emits will be the fifth of the first.

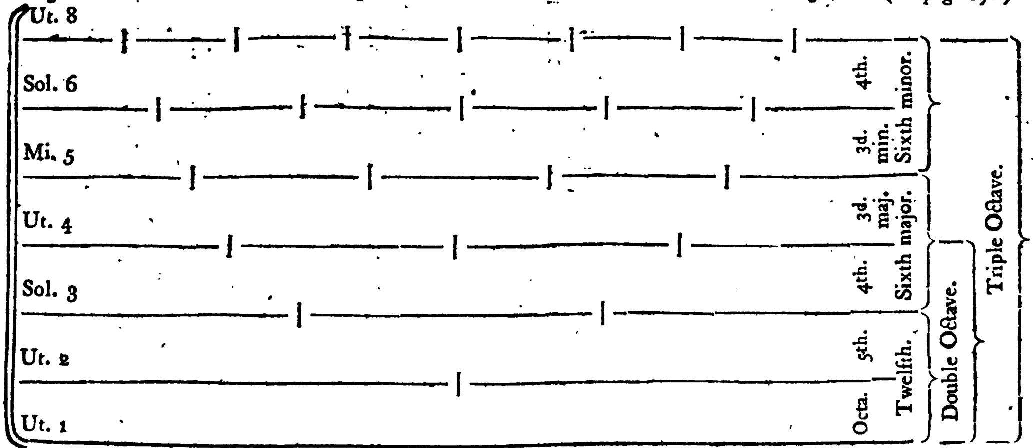
If the length be reduced to three-fourths, it will give the fourth of the first.

If it be reduced to $\frac{4}{5}$, it will give the third major; if to $\frac{5}{6}$, the third minor. If reduced to $\frac{8}{9}$, it will give what is called the tone major; if to $\frac{9}{10}$, the tone minor; and if to $\frac{15}{16}$, the semi-tone, or that which in the gamut is between *mi* and *fa* or *si* and *sol*.

The same results will be obtained if a string be fastened at both ends, and $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of it be successively intercepted by means of a moveable bridge.

(See the following table relating to this subject.)

Ingenious Manner in which Rameau expresses the Relation of the Sounds in the Diatonic Progression. (*See page* 198.)



It may be here seen that if these seven lines represent seven strings, of equal length, the order of the principal harmonic concords will be determined by the following numbers:

Thus,								
1 to 2	denotes	The octave	4 to 5	denotes	The third major	3 to 5	denotes	The sixth major
2 to 3		The fifth	5 to 6		The third minor	5 to 8		The sixth minor
3 to 4		The fourth	6 to 8		The fourth	1 to 4		The double octave
			1 to 3		The twelfth	1 to 8		The triple octave

Such is the reſult of a determinate degree of tenſion applied to a ſtring, when the length of it has been made to vary. Let us now ſuppoſe that the length of the ſtring is conſtantly the ſame, but that its degree of tenſion is varied. The following is what we are taught by experiment on this ſubject:

If a weight be ſuſpended at one end of a ſtring of a determinate length, made faſt by the other, and if the tone it emits be fixed, when another weight quadruple of the former is applied, the tone will be the octave of the former; if the weight be nine times as heavy, the tone will be the octave of the fifth; and ſo on; ſo that the tones will become acute in the ratio of the ſquare roots of the weights.

The ſize of the ſtrings has an effect in regard to the tones, as well as the different lengths of the ſtring, and the weight by which it is ſtretched; for it is proved by experiment, that a ſtring twice as ſmall in diameter as another, every thing elſe being the ſame, emits a tone which is the octave of that of the other; and that if the diameter is only a third of that of the other, the tone is the octave of the fifth of that other ſtring, following the order of the diatonic ſcale.

We may thence conclude, that the tones of the muſical ſtrings are in the direct ratio of the ſquare root of the weights by which they are ſtretched, and in the inverſe ratio of the lengths and diameters of theſe ſtrings.

Conſequently, to bring into uniſon ſtrings which differ in length and diameter, and which are ſtretched by different weights, the compound ratio thence reſulting muſt be exactly the ſame, in order that the frequency of the vibrations in one may be compenſated by the ſlowneſs of another. Thus two ſtrings of the ſame ſize, the lengths of which are as 2 to 1, and the ſtretching weights as 4 to 1, will have their vibrations iſochronous, that is to ſay, they will be in uniſon: two ſtrings, the diameters of which are as 2 to 1, and the lengths as 1 to 2, ſtretched by equal weights, will be in uniſon alſo, as well as thoſe which, being of equal lengths, have their diameters as 2 to 1, and the ſtretching weight as 4 to 1.

We may conclude therefore, that two ſtrings, the diameters of which are as 3 to 2, and the lengths as 1 to 3, cannot be in uniſon, unleſs the weights, by which they are ſtretched, be to each other in the ſame ratio as 1 to 4.

To determine the number of the vibrations made by a ſtring of a given length and ſize, when ſtretched by a given weight.

A very ingenious method, invented by M. Sauveur, for finding the number of theſe vibrations, may be ſeen in the Memoirs of the Academy of Sciences, for 1700. Having obſerved, when two organ-pipes, very low and having tones very near to each other, were founded at the fame time, that a feries of pulfations or beats were heard in the founds; and by reflecting on the caufe of this phenomenon, he found that thefe beats arofe from the periodical meeting of the coincident vibrations of the two pipes. Hence he concluded, that if the number of thefe pulfations, which took place in a fecond, could be afcertained by a ftop watch, and if it were poffible alfo to determine, by the nature of the confonance of the two pipes, the ratio of the vibrations which they made in the fame time, he fhould be able to afcertain the real number of the vibrations made by each.

We fhall here fuppofe, for example, that two organ-pipes are exactly tuned, the one to *mi* flat, and the other to *mi*: it is well known, that as the interval between thefe two tones is a femi-tone minor, expreffed by the ratio of 24 to 25, the higher pipe will perform 25 vibrations while the lower performs only 24; fo that at each 25th vibration of the former, or the 24th of the latter, there will be a pulfation; if 6 pulfations therefore are obferved in the courfe of 1 fecond, we ought to conclude, that 24 vibrations of the one and 25 of the other take place in the tenth of a fecond: and confequently that the one performs 240 vibrations, and the other 250, in the courfe of a fecond.

M. Sauveur made experiments according to this idea, and found that an open organ-pipe, 5 feet in length, makes 100 vibrations per second; consequently, one of 4 feet, which gives the lower triple octave, and the lowest sound perceptible to the ear, would make only $12\frac{1}{2}$; on the other hand, a pipe of one inch less $\frac{1}{16}$, being the shortest the sound of which can be distinguished, will give in a second 6400 vibrations. The limits, therefore, of the slowest and the quickest vibrations appreciable by the ear, are according to M. Sauveur, $12\frac{1}{2}$ and 6400.

We shall not enlarge further on these details, but proceed to a very curious phenomenon respecting strings in a state of vibration.

Make fast a string by both its extremities, and by means of a bridge divide it into aliquot parts, for example, 3 on the one side and 1 on the other, and put the larger part, that is to say the $\frac{3}{4}$ in a state of vibration; if the bridge absolutely intercepts all communication from the one part to the other, these $\frac{3}{4}$ of the string, as is well known, will give the tone of the fourth of the whole string; if $\frac{4}{5}$ be intercepted, the tone will be the tierce major.

But if this bridge only prevents the whole of the string from vibrating, without intercepting the communication of motion from the one part to the other, the greater part will then emit only the same sound as the less, and the $\frac{3}{4}$ of the string, which in the former case gave the fourth of the whole string, will give only the double octave, which is the tone proper to the fourth of the ſtring. The caſe is the ſame if this fourth be touched: its vibrations, by being communicated to the other three-fourths, will make them ſound, but in ſuch a manner as to give only this double octave.

The following reaſon, which may be rendered plain by an experiment, is aſſigned for this phenomenon: when the bridge abſolutely intercepts all communication between the two parts of the ſtring, the whole of the largeſt part vibrates together; and if it be $\frac{3}{4}$ of the whole ſtring, it makes, agreeably to the general law, 4 vibrations in the time that the whole ſtring would make 3: its ſound therefore is the fourth of the whole ſtring.

But in the ſecond caſe, the larger part of the ſtring divides itſelf into 3 aliquot parts, each of which is equal to the leſs, and all theſe diſtinct portions perform their particular vibrations; for if bits of red paper, for example, be placed upon all the points of diviſion, and bits of white paper in the middle of each diviſion, the former will remain motionleſs, but the latter will drop off as ſoon as the ſtring begins to vibrate.

If the part of the ſtring immediately made to vibrate, inſtead of an aliquot part of the remainder, be only $\frac{2}{7}$ of it, the whole ſtring will then divide itſelf into ſeventh parts, and will emit only that tone which belongs to $\frac{1}{7}$ of its length.

If the leſs part of the ſtring be incommenſurable to the greater, the found will abfolutely be difcordant, and will almoft immediately ceafe.

Method of adding, fubtracting, multiplying, and dividing concords.

It is neceffary for thofe who wifh to underftand the theory of mufic, to know what concords refult from two or more concords, either when added or fubtracted, or when multiplied by each other. For this reafon we fhall give the following rules:

PROB. I. *To add one concord to another.*

Exprefs the two concords by the fractions which reprefent them, and then multiply thefe two fractions together: that is to fay, firft the numerators and then the denominators: the number thence produced will exprefs the concord refulting from the fum of the two concords given.

EXAMPLE I.

Let it be required to add the fourth and fifth together.

The expreffion for the fifth is $\frac{2}{3}$, and that for the fourth $\frac{3}{4}$, the product of which is $\frac{6}{12} = \frac{1}{2}$, being the expreffion for the octave. It is indeed well known that the octave is compofed of a fifth and a fourth.

EXAMPLE II.

What is the concord arising from the addition of the third major and the third minor?

The expression of third major is $\frac{4}{5}$, and that of the third minor is $\frac{5}{6}$, the product of which is $\frac{20}{30}$ or $\frac{2}{3}$, which expresses the fifth; and this concord indeed is composed of a third major and a third minor.

EXAMPLE III.

What is the concord produced by the addition of two tones major?

A tone major is expressed by $\frac{8}{9}$; consequently, to add two tones major, $\frac{8}{9}$ must be multiplied by $\frac{8}{9}$. The product $\frac{64}{81}$ is a fraction less than $\frac{64}{80}$ or $\frac{4}{5}$, which expresses the third major; hence it follows, that the concord expressed by $\frac{64}{81}$ is greater than the third major, and consequently two tones major are more than a third major, or form a third major false by excess.

On the other hand, by adding two tones minor, which are each expressed by $\frac{9}{10}$, it will be found that their sum $\frac{81}{100}$ is greater than $\frac{80}{100}$ or $\frac{4}{5}$, which denotes the third major: two tones minor therefore added together make more than a third major.

This third is indeed composed of a tone major and a tone minor, as may be proved by adding together the concords $\frac{8}{9}$ and $\frac{9}{10}$, which make $\frac{72}{90}$ = $\frac{8}{10}$ or $\frac{4}{5}$.

It might be proved, in like manner, that two semi-tones major make more than a tone major, and two semi-tones minor less even than a tone minor; and, in the last place, that a semi-tone major and a semi-tone minor make exactly a tone minor.

PROB. II. *To subtract one concord from another.*

Instead of multiplying together the fractions which express the given concords, invert that which expresses the concord to be subtracted from the other, and then multiply them together as before: the product will give a fraction expressing the concord required.

EXAMPLE I.

What is the concord which results from the fifth subtracted from the octave?

The expression of the octave is $\frac{1}{2}$; that of the fifth $\frac{2}{3}$, which inverted gives $\frac{3}{2}$; and if $\frac{1}{2}$ be multiplied by $\frac{3}{2}$ we shall have $\frac{3}{4}$, which expresses the fourth.

EXAMPLE II.

What is the difference between the tone major and the tone minor?

The tone major is expressed by $\frac{8}{9}$, and the tone minor by $\frac{9}{10}$, which when inverted gives $\frac{10}{9}$: the

the product of $\frac{8}{9}$ by $\frac{10}{9}$ is $\frac{80}{81}$, which expresses the difference between the tone major and the tone minor. This is, what is called the *great comma*.

PROB. III. *To double a concord, or to multiply it any number of times, at pleasure.*

In this case, nothing is necessary but to raise the terms of the fraction, which expresses the given concord, to the power denoted by the number of times it is to be multiplied; that is, to the square if it is to be doubled, to the cube if to be tripled, and so on.

Thus, the concord arising from the tone major tripled, is $\frac{512}{729}$; for as the expression of the tone major is $\frac{8}{9}$, we shall have $8 \times 8 \times 8 = 512$, and $9 \times 9 \times 9 = 729$. This concord $\frac{512}{729}$ corresponds to the interval between *ut* and a *fa*, higher than *fa* sharp of the gamut.

PROB. IV. *To divide one concord by any number at pleasure, or to find a concord which shall be the half, third, &c. of a given concord.*

To answer this problem, take the fraction which expresses the given concord, and extract that root of it which is denoted by the determinate divisor; that is to say, the square root, if the concord is to be divided into two; the cube root, if it is to be divided into three, &c.; and this root will expreſs the concord required.

EXAMPLE.

As the octave is expreſſed by $\frac{1}{2}$, if the ſquare root of it be extracted, it will give $\frac{7}{10}$ nearly; but $\frac{7}{10}$ is leſs than $\frac{3}{4}$, and greater than $\frac{2}{3}$; conſequently the middle of the octave is between the fourth and the fifth, or very near *fa* ſharp.

Of the reſonance of ſonorous bodies, the fundamental principle of harmony and melody, with ſome other harmonical phenomena.

EXPERIMENT I.

If you liſten to the ſound of a bell, eſpecially when very grave, however indifferent your ear may be, you will eaſily diſtinguiſh, beſides the principal ſound, ſeveral other ſounds more acute; but if you have an ear accuſtomed to appreciate the muſical intervals, you will perceive that one of theſe ſounds is the twelfth or fifth above the octave, and another the ſeventeenth major or the third major above the double octave. If your ear be exceedingly delicate, you will diſtinguiſh alſo its octave, its double and even its triple octave: the latter indeed are ſomewhat more difficult to be heard, becauſe the octaves are almoſt confounded with the fundamental ſound, in conſequence of that natural ſenſation which makes us confound the octave with uniſon.

If the bow of a violoncello be strongly rubbed against one of its large strings, or the string of a trumpet marine, you will perceive the same effect. In a word, if you have an experienced ear, you will be able to distinguish these different sounds, either in the resonance of a string or in that of any other sonorous body, and even in the voice.

Another method of making this experiment.

Suspend a pair of tongs by a woolen or cotton cord, or any other kind of small string, and twisting the extremities of it around the fore-finger of each hand, put these two fingers into your ears. If the lower part of the tongs be then struck, you will first hear a loud and grave sound, like that of a large bell at a distance; and this tone will be accompanied by several others, more acute, among which, when they begin to die away, you will distinguish the twelfth and the seventeenth of the lowest tone. Rameau confirmed the truth of this phenomenon by the help of several organ-pipes.

This experiment respecting the resonance of sonorous bodies, is not new. It was known to Dr. Wallis and to Mersenne, who speak of it in their works; but it appeared to them a simple phenomenon, with the consequences of which they were entirely unacquainted. Rameau first discovered the use of it, in deducing all the rules of musical composition, which before had been founded on mere ſentiment, and on experience, incapable of ſerving as a guide in all caſes, and of accounting for every effect. It forms the baſis of his theory of thorough baſs; a ſyſtem which has been oppoſed with much declamation, but which, however, moſt muſicians ſeem at preſent to have adopted.

All his harmony therefore is multiple, and compoſed of ſounds which would give the aliquot parts of the ſonorous body, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and we might add $\frac{1}{7}$, $\frac{1}{8}$, &c. But the weakneſs of theſe ſounds, which go on always decreaſing in ſtrength, renders it difficult to diſtinguiſh them. Rameau however ſays, that he could diſtinguiſh very plainly the ſound expreſſed by $\frac{1}{7}$, which is the double octave of a ſound divided nearly into two equal parts, being the interval between *la* and *si* flat, below the firſt octave: he calls it a loſt ſound, and totally excludes it from harmony.

EXPERIMENT II.

If you adjuſt ſeveral ſtrings to the octave, the twelfth, and the ſeventeenth, of the determinate ſound emitted by another ſtring, both aſcending and deſcending; as often as you make that which gives the determinate ſound to reſound ſtrongly, you will immediately ſee all the reſt put themſelves in a ſtate of vibration: you will even hear thoſe ſound which are tuned lower, if you take care to damp suddenly, by means of a soft body, the sound of the former.

Most people have heard the glasses on a table emit a sound when a person near them has been singing with a strong and a loud voice. The strings of an instrument, though not touched, are often heard to sound in consequence of the same cause, especially after swelling notes long continued.

In a manner somewhat similar, the diversity of tones agitates, in various ways, the fibres of our bodies, excites the passions, and produces in the soul sensations so different.

On the harmonical sounds heard with the principal sound: Whether they have their source immediately in the sonorous body, or exist in the air or the organ?

It is very probable that the principal sound is the only one that derives its origin immediately from the vibrations of the sonorous body. Philosophers of eminence have endeavoured to discover whether, independently of the total vibrations made by a body, there are also partial vibrations; but hitherto they have been able to observe only simple vibrations. Besides, how can it be conceived that the whole of a string should be in vibration, and that during its motion it should divide itself into two or three parts that perform also their distinct vibrations?

It muſt then be ſaid, that theſe harmonical ſounds of octave, twelfth, ſeventeenth, &c. are in the air, or the organ: both ſuppoſitions are probable; for ſince a determinate ſound has the property of putting into a ſtate of vibration bodies diſpoſed to give its octave, its twelfth, &c. we muſt allow that this ſound may put in motion the particles of the air, ſuſceptible of vibrations of double, triple, quadruple, and quintuple velocity. What however appears moſt probable in this reſpect, is, that theſe vibrations exiſt only in the ear: it ſeems indeed to be proved by the anatomy of this organ, that ſound is tranſmitted to the ſoul only by the vibrations of thoſe nervous fibres which cover the interior part of the ear; and as they are of different lengths, there are always ſome of them which perform their vibrations iſochronous to thoſe of a given ſound. But, at the ſame time, and in conſequence of the property above mentioned, this ſound muſt put in motion thoſe fibres ſuſceptible of iſochronous vibrations, and even thoſe which can make vibrations of double, triple, quadruple, &c. velocity. Such, in our opinion, is the moſt probable explanation that can be given of this ſingular phenomenon.

Of the modern muſic.

Every one knows that the gamut, or diatonic ſcale, is repreſented by the ſounds *ut*, *re*, *mi*, *fa*, *sol*, *la*, *si*, *ut*, which complete the whole extent of the octave; and it appears, from the generation of it, as explained by Rameau, that from *ut* to *re* there is a tone major; from *re* to *mi* a tone minor; from *mi* to *fa* a semi-tone major; from *fa* to *sol* a tone major, as well as from *sol* to *la*; and, in the last place, that from *la* to *si* there is a tone minor, and from *si* to *ut* a semi-tone major.

It is thence concluded, that in this scale there are three intervals which are not entirely just: these are:

1st. The third minor, from *re* to *fa*, which being composed of a tone minor and a semi-tone major, is only in the ratio of 27 to 32; but this ratio is somewhat less than that of 5 to 6, which expresses exactly the third minor.

2d. The third major, from *fa* to *la*, is too high, being composed of two tones major; whereas, to be exactly in the ratio of 4 to 5, it ought to consist only of a tone major and a tone minor.

3d. The third minor, from *la* to *ut*, is as far from being just as that from *re* to *fa*, and for the same reason.

On the cause of the pleasure arising from music.— The effects of harmony on man and on animals.

It has often been asked, why two sounds, which form together the fifth and the third, excite pleafure, while the ear experiences a difagreeable fenfation, by hearing founds which are no more than a tone or a femi-tone diftant from each other. Though it is difficult to anfwer this queftion, the following obfervations may tend to throw fome light upon it.

Pleafure, we are told, arifes from the perception of relations, as may be proved by various examples taken from the arts. The pleafure therefore derived from mufic, confifts in the perception of the relations of founds. But are thefe relations fufficiently fimple for the foul to perceive and diftinguifh their order? Sounds will pleafe when heard together in a certain order; but, on the other hand, they will difpleafe if their relations are too complex, or if they are abfolutely deftitute of order.

This reafoning will be fufficiently proved by an enumeration of the known concords and difcords. In unifon, the vibrations of two founds continually coincide, throughout the whole time of their duration; this is the fimpleft kind of relation. Unifon alfo is the firft concord in the octave; the two founds of which it is compofed perform their vibrations in fuch a manner, that two of the one are completed in the fame time as one of the other: thus the unifon is fucceeded by the octave. It is fo natural to man, that he, who through fome defect in his voice, cannot reach a found too grave or too acute, falls into the higher or lower octave.

When the vibrations of two sounds are performed in such a manner, that three of the one correspond to one of the other, these give the simplest relation next to those above-mentioned. Who does not know that the concord most agreeable to the ear is the twelfth, or the octave of the fifth? In that respect it even surpasses the fifth.

Next to the fifth, is the double octave of the fifth, or the seventeenth major, which is expressed by the ratio of 1 to 3. This concord, next to the twelfth, is the most agreeable.

The fourth, expressed by $\frac{3}{4}$, the third minor, expressed by $\frac{5}{6}$, and the sixths, both major and minor, expressed by $\frac{5}{8}$ and $\frac{3}{5}$, are concords, for the same reason.

But it appears that all the other sounds, after these relations, are too complex for the soul to perceive their order.

The following very strong objection, however, may be made to this reasoning. How can the pleasure arising from concords consist in the perception of them, since the soul often does not know whether such relations exist between the sounds? The most ignorant person is no less pleased with an harmonious concert than he who has calculated the relation of all its parts: what has hitherto been ſaid, may therefore be more ingenious than ſolid.

We cannot help acknowledging, that we are rather inclined to think ſo; and it appears to us, that the celebrated experiment on the reſonance of bodies, may ſerve to account, in a ſtill more plauſible manner, for the pleaſure ariſing from concords; becauſe, as every ſound degenerates into mere noiſe when not accompanied with its twelfth and its ſeventeenth major, beſides its octaves, is it not evident, that when we combine any ſound with its twelfth, or its ſeventeenth major, or with both at the ſame time, we only imitate the proceſs of nature, by giving to that ſound, in a fuller and more ſenſible manner, the accompaniment which nature itſelf gives it, and which cannot fail to pleaſe the ear, on account of the habit it has acquired of hearing them together? This is ſo agreeable to truth, that there are only two primitive concords, the twelfth and the ſeventeenth major; and that the reſt, as the fifth, the third major, the fourth and the ſixth are derived from them. We know alſo, that theſe two primitive concords, are the moſt perfect of all, and that they form the moſt agreeable accompaniment that can be given to any ſound, though, on the harpſichord for example, to facilitate the execution, the third major and the fifth itſelf, which, with the octave, form what is called perfect harmony, are ſubſtituted in their ſtead. But this harmony is perfect only by repreſentation, and the moſt perfect of all, would be that in which the twelfth and the ſeventeenth were combined with the fundamental found and its octaves. Rameau therefore adopted it as often as he could in his choruſes. We might enlarge farther on this idea; but what has been already ſaid will be ſufficient for every intelligent reader.

Some very extraordinary things are related in regard to the effects produced by the muſic of the ancients, which on account of their ſingularity we ſhall here mention. We ſhall then examine them more minutely, and ſhew that, in this reſpect, the modern muſic is not inferior to the ancient.

Agamemnon, it is ſaid, when he ſet out on the expedition againſt Troy, being deſirous to ſecure the fidelity of his wife, left with her a Dorian muſician, who by the effect of his airs rendered fruitleſs, for a long time, the attempts of Ægiſthus to obtain her affection; but that prince having diſcovered the cauſe of her reſiſtance, got the muſician put to death, after which he triumphed, without difficulty, over the virtue of Clytemneſtra.

We are told alſo that, at a later period, Pythagoras compoſed ſongs or airs capable of curing the moſt violent paſſions, and of recalling men to the paths of virtue and moderation: While the phyſician preſcribes draughts for curing bodily diſeaſes, an able musician might therefore prescribe an air for rooting out a vicious passion.

The story of Timotheus, the director of the music of Alexander the Great, is well known. One day, while the prince was at table, Timotheus performed an air in the Phrygian taste, which made such an impression on him, that being already heated with wine, he flew to his arms and was going to attack his guests, had not Timotheus immediately changed the stile of his performance to the Sub-Phrygian. This change calmed the impetuous fury of the monarch, who resumed his place at table. This was the same Timotheus who at Sparta experienced the humiliation of seeing publicly suppressed four strings which he had added to his lyre. The severe Spartans thought that this innovation would tend to effeminate their manners, by introducing a more extensive and more variegated kind of music. This at any rate proves, that the Greeks were convinced that music had a peculiar influence on manners; and that it was the duty of government to keep a watchful eye over that art.

Who indeed can doubt that music is capable of producing such an effect? Let us only interrogate ourselves, and examine what have been our sensations on hearing a majestic or warlike piece of music, or a tender and pathetic air sung or played with expression. Who does not feel that the latter tends as much to melt the ſoul and diſpoſe it to pleaſure, as the former to rouſe and exalt it? Several facts in regard to the modern muſic place it, in this reſpect, on a level with the ancient.

The modern muſic indeed has had alſo its Timotheus, who could excite or calm at his pleaſure the moſt impetuous emotions. Henry III. king of France, having given a concert on occaſion of the marriage of the Duke de Joyeuſe, Claudin le Jeune, a celebrated muſician of that period, executed certain airs, which had ſuch an effect on a young nobleman, that he drew his ſword and challenged every one near him to combat; but Claudin, equally prudent as Timotheus, inſtantly changed to an air apparently Sub-Phrygian, which appeaſed the furious youth.

What ſhall we ſay of Stradella, the celebrated compoſer, whoſe muſic made the daggers drop from the hands of his aſſaſſins? Stradella having carried off the miſtreſs of a Venetian muſician, and retired with her to Rome, the Venetian hired three deſperadoes to aſſaſſinate him; but fortunately for Stradella they had an ear ſenſible to harmony. Theſe aſſaſſins, while waiting for a favourable opportunity to execute their purpoſe, entered the church of St. John de Latran, during the performance of an oratorio, compoſed by the perſon whom they intended to deſtroy, and were ſo affected by the muſic that they abandoned their deſign, and even waited on the muſician to forewarn him of his danger. Stradella, however was not always ſo fortunate; other aſſaſſins, who apparently had no ear for muſic, ſtabbed him ſome time after at Genoa: this event took place about the year 1670.

Every body almoſt has heard that muſic is a cure for the bite of the tarantula. This cure, which was formerly conſidered as certain, has by ſome been conteſted; but, however this may be, Father Schott in his works gives the tarantula air, which appears to be very dull, as well as that employed by the Sicilian fiſhermen to entice the thunny fiſh into their nets. But it is probable that fiſh are no great connoiſſeurs in muſic.

Various anecdotes are related reſpecting perſons whoſe lives have been preſerved by muſic effecting a ſort of revolution in their conſtitutions. A woman being attacked, for ſeveral months, with the vapours, and confined to her apartment, had reſolved to ſtarve herſelf to death: ſhe was however prevailed on, but not without great difficulty, to ſee a repreſentation of the *Serva Padrona*, at the concluſion of which ſhe found herſelf almoſt cured; and renouncing her melancholy reſolution, was entirely reſtored to health by a few more repreſentations of the like kind.

There is a celebrated air in Switzerland, called *Ranz des Vaches*, which had ſuch an extraordinary effect on the Swiſs troops in the French ſervice, that they always fell into a deep melancholy when they heard it; Louis XIV. therefore, forbade it ever to be played in France, under the pain of a severe penalty. We are told of a Scotch air, which has a similar effect on the natives of Scotland.

Most animals and even insects are not insensible to the pleasure of music. There are few musicians perhaps who have not seen spiders suspend themselves by their threads in order to be near the instruments. We have several times had that satisfaction. We have seen a dog, who at the adagio of a sonata never failed to shew signs of attention, and some peculiar sensation by howling.

The most singular fact, however, is that mentioned by Burney in his History of Music. This author relates, that an officer being shut up in the Bastille, had permission to carry with him a lute, on which he was an excellent performer; but he had scarcely made use of it for three or four days, when the mice issuing from their holes, and the spiders suspending themselves from the cieling by their threads, assembled around him to participate in his melody. His aversion to these animals made their visit at first disagreeable; and induced him to lay aside this recreation; but he soon was so accustomed to them, that they became a source of amusement.

We have learned from persons worthy of credit, now in London, that during their residence in the

Levant, they have witneſſed the influence of certain Greek ſongs on the oxen, which the Greek farmers employ in agriculture.

Thoſe who have ſeen, at Bartholomew fair, in Smithfield, two elephants follow exactly the meaſure of the tunes played at the entrance of the place where they were kept, and humour all their variations, by the motion of their head and trunk, will find no difficulty in believing what Buffon has ſaid reſpecting the ſingular taſte of theſe animals for harmony.

In a word, without deciding whether the fables of Amphion and Arion may not, in ſome meaſure be founded on truth, we know that the noiſy ſound of trumpets, and the harmony of military inſtruments, excite the courage of ſoldiers, and the ardour of horſes; and the directors of caravans take care to be accompanied on their march, by performers on different inſtruments, the muſic of which has ſuch an effect on their camels, that they are better enabled to ſuſtain the fatigue they muſt undergo in traverſing the burning deſarts of Arabia or Africa.

Of the properties of certain inſtruments, and particularly wind inſtruments.

We are perfectly well acquainted with the manner in which ſtringed inſtruments emit their ſounds; but erroneous ideas were long entertained in regard to wind inſtruments, ſuch as the flute; for the ſound was aſcribed to the interior ſurface of the tube. The celebrated Euler firſt rectified this error, and it reſults from his reſearches:

1ſt. That the ſound, produced by a flute, is nothing elſe than that of the cylinder of air contained in it.

2d. That the weight of the atmoſphere, which compreſſes it, acts the part of a ſtretching weight.

3d. That the ſound of this cylinder of air, is exactly the ſame as that which would be produced by a ſtring of the ſame maſs and length, extended by a weight equal to that which compreſſes the baſe of the cylinder.

This fact is confirmed by experiment and calculation; for Euler found that a cylinder of air of $7\frac{1}{2}$ Rhinlandiſh feet, at a time when the barometer is at a mean height, muſt give c-*ſol-ut*; and ſuch is nearly the length of the open pipe of an organ which emits that ſound. The reaſon of its being generally made 8 feet, is becauſe that length is required at thoſe times when the weight of the atmoſphere is greater.

Since the weight of the atmoſphere produces, in regard to the ſounding cylinder of air, the ſame effect as that produced by the weight which ſtretches a ſtring, the more that weight is increaſed, the more will the ſound be elevated; it is therefore obſerved that during ſerene, warm weather, the tone of wind inſtruments is raiſed; and that during cold and ſtormy weather it is lowered. Theſe inſtruments alſo become higher in proportion as they are heated; becauſe the maſs of the cylinder of heated air becoming leſs, while the weight of the atmoſphere remains unchanged, the caſe is exactly the ſame as if a ſtring ſhould become leſs, and be ſtill ſtretched by the ſame weight: every body knows that ſuch a ſtring would emit a higher tone.

But as ſtringed inſtruments muſt become lower, becauſe the elaſticity of the ſtrings inſenſibly decreaſes, it thence follows that wind and ſtringed inſtruments, however well tuned they may be to each other, ſoon become diſcordant.

A very ſingular phenomenon is obſerved in regard to wind inſtruments, ſuch as the flute and huntſman's horn: With a flute, for example, when all the holes are ſtopped, if you blow faintly into the mouth aperture, a certain tone will be produced; if you blow a little ſtronger, the tone inſtantly riſes to the octave, and by blowing ſucceſſively with more force, you will produce the twelfth or fifth above the octave; then the double octave or ſeventeenth major.

Of ſome muſical inſtruments, or machines, remarkable for their ſingularity or conſtruction.

At the head of all theſe muſical inſtruments, or machines, we ought doubtleſs to place the organ;

the extent and variety of the tones of which would excite much more admiration if it were not ſo common as it is in our churches; for beſides the artifice neceſſary to produce the tones by means of keys, what ingenuity muſt have been required to contrive mechaniſm for giving that variety of character to the tones obtained by means of the different ſtops, ſuch as thoſe called the voice ſtop, flute ſtop, &c.? A complete deſcription, therefore of an organ, and of its conſtruction, would be ſufficient to occupy a large volume.

The ancients had hydraulic organs, that is to ſay, organs the ſound of which was occaſioned by air produced by the motion of water. Theſe machines were invented by Cteſibius of Alexandria, and his ſcholar Hero. From the deſcription of theſe hydraulic organs, given by Vitruvius, in the tenth book of his architecture, Perrault conſtructed one which he depoſited in the king's library, where the Royal Academy of Sciences held their ſittings. This inſtrument indeed is not to be compared to the modern organs; but it is evident that the mechaniſm of it has ſerved as a baſis for that of ours. St. Jerome ſpeaks with enthuſiaſm of an organ which had twelve pair of bellows, and which could be heard at the diſtance of a mile. It thence appears that the method employed by Cteſibius, to produce air to fill the wind-box, was ſoon laid aſide for one more ſimple; that is to ſay, for a pair of bellows.

The performer on the *tambour de basque*, and the automaton flute player of Vaucanson, which were exhibited and seen with admiration in most parts of Europe, in the year 1749, may be classed among the most curious musical machines ever invented. We shall not however say any thing of the former of these machines, because the latter appears to have been far more complex.

The automaton flute-player performed several airs on the flute, with the precision and correctness of the most expert musician. It held the flute in the usual manner, and produced the tone by means of its mouth; while its fingers, applied on the holes, produced the different notes. It is well-known how the fingers might be raised by spikes fixed in a cylinder, so as to produce these sounds; but it is difficult to be conceived how that part could be executed which is performed by the tongue, and without which the music would be very defective. Vaucanson indeed confesses, that this motion in his machine, was that which cost him the greatest labour.

A very convenient instrument for composers, invented in Germany, consists of a harpsichord, which by certain machinery added to it, notes down any air while a person is playing it. This is a great advantage to composers, as it enables them, when hurried away by the fervor of their imagination, to preferve what has fucceffively received from their fingers a fleeting exiftence, and what otherwife it would often be impoffible for them to remember. A defcription of this machine may be found in the Memoirs of the Academy of Berlin for the year 1773.

Of a new inftrument called the Harmonica.

This inftrument was invented in America by Dr. Franklin, who gave a difcription of it to father Beccaria, which the latter publifhed in his works, printed in 1773.

It is well known that when the finger, a little moiftened, is rubbed againft the edge of a drinking glafs, a fweet found is produced; and that the tone varies according to the form, fize, and thicknefs of the glafs. The tone may be raifed or lowered alfo by putting into the glafs a greater or lefs quantity of water. Dr. Franklin fays that an Irifhman, named Puckeridge, firft conceived the idea, about twenty years before, of conftructing an inftrument with feveral glaffes of this kind, adjufted to the various tones, and fixed to a ftand in fuch a manner, that different airs could be played upon them. Mr. Puckeridge having afterwards been burnt in his houfe, along with this inftrument, Mr. Delaval conftructed another of the fame kind, with glaffes better chofen, which he applied to the like purpofe. About fourteen or fifteen years ago, an Englifh lady at Paris, performed, it is ſaid, exceedingly well on this inſtrument, which however did not long continue in vogue: at preſent it is confined to cabinets among other muſical curioſities.

A juggler, ſome years ago, to ſhew his dexterity, placed on a table eight glaſſes of the ſame ſize, which had all the ſame tone, and boaſted that he could tune them in an inſtant by pouring water into them, ſo as to play an air with the utmoſt preciſion. "Thoſe who tune violins or organs," ſaid he, "are not ſo dexterous as I; ſince they often labour for a quarter of an hour, and try the ſame pipe or ſtring twenty times, before they can bring it to the proper tone." While he pronounced theſe words he poured water into the eight glaſſes; then ſtriking them one after the other with a ſmall rod, he immediately ſhewed that they emitted with great exactneſs the tones of the gamut, *ut*, *re*, *mi*, *fa*, *ſol*, *la*, *ſi*, *ut*; and as he then amuſed the ſpectators by playing an air, which he accompanied with his voice, they overlooked the artifice he had employed in tuning his inſtrument ſo ſpeedily.

Each of the glaſſes had a ſmall hole at the proper height, ſo that when filled to the brim the water ran out till there remained no more than the quantity requiſite to give the glaſs the neceſſary tone. By theſe means, the inſtrument tuned itſelf in a moment; and the muſician had no occaſion to pour in or pour out water, at different times, to render the tone graver or more acute.

On what is called a false voice.

A fine voice is certainly preferable to every instrument whatever. Unfortunately, many persons have only a false voice; but, in general, this does not arise from any defect in the organs of the voice, which are almost the same in all mankind: it originates from the ears, owing to an inequality of strength in these organs, or to some want of delicacy or tension, in consequence of which, as they receive unequal impressions, we necessarily hear false sounds, and the voice, which endeavours to imitate them, becomes itself false. On this subject Dr. Vandermonde made a very simple experiment, which he relates in his Essay on Improving the Human Mind, and which may be repeated on children who pronounce with a false voice, in order that a remedy may be applied at that tender age when the organs are still susceptible of modification.

The experiment, as he describes it, is as follows: "I made choice," says he, "of a clear day, and having fixed on a spacious apartment, I took up my station in a place judged most convenient for my experiments. I then stopped one of the ears of the child who was to be the subject of them, and made her recede from me, till she no longer heard the found of a repeating watch which I held in my hand, or at leaft until the found of the bell produced a very weak impreffion on her organs of hearing. I then defired her to remain in that place, and immediately going up to her unftopped her ear, and ftopped the other, taking care to caufe her to fhut her mouth, left the found fhould be communicated to the ear through the euftachian tube. I then returned to my ftation, and making my watch again ftrike, the child was quite furprifed to find that fhe heard tolerably well; upon which I made a fign to her to recede again till fhe could fcarcely hear the found." It refults from this experiment, that in the ears of perfons who have a falfe voice, there is an inequality of ftrength, and the means of remedying this defect in children, is to afcertain by a fimilar mode, which ear is the weakeft. "When this has been difcovered, nothing better can be done, in my opinion," fays Dr. Vandermonde, "than to ftop up the other as much as poffible, and to take advantage of that valuable opportunity of frequently exercifing the weak ear, but in fuch a manner as not to fatigue it. The one thus made to labour alone will be ftrengthened, while the other will always retain the fame force. The child's ear fhould from time to time be unftopped, in order to make it fing, and to difcover whether both ears have the fame degree of fenfibility." This natural defect may be then corrected, and any perfon may be made to acquire a true voice, provided the means pointed out by Dr. Vandermonde be early employed.

Perſons who have a falſe voice, in conſequence of ſome inequality in the ears, may be compared to thoſe who ſquint; that is to ſay who, in order to ſee an object diſtinctly, do not turn equally towards it the axis of both eyes, becauſe they have not the ſame viſual powers. It is probable that the former, if they had early accuſtomed themſelves to make uſe of only one ear, would hear diſtinctly different ſounds which they would have imitated, and would not have contracted a falſe voice.

Of the Speaking Trumpet, and Ear Trumpet.

As the ſight is aſſiſted by teleſcopes and microſcopes, ſimilar inſtruments have been deviſed alſo for aſſiſting the faculty of hearing. One of theſe, called the ſpeaking trumpet, is employed for conveying ſound to a great diſtance: the other, called the ear trumpet, ſerves to magnify to the ear the leaſt whiſper.

Sir Thomas Morland, among the moderns, beſtowed the moſt labour in endeavouring to improve this method of enlarging and conveying ſound; and on this ſubject he publiſhed a treatiſe, entitled *De Tubâ Stentorophonica*, a name which alludes to the voice of Stentor, ſo celebrated among the Greeks for its great ſtrength. The following obſervations on this ſubject are in part borrowed from that curious work.

The ancients, it would ſeem, were acquainted with the ſpeaking trumpet, for we are told that Alexander had a horn, by means of which he could give orders to his whole army, however numerous. Kircher, on the authority of ſome paſſages in a manuſcript, preſerved in the Vatican, makes the diameter of its greateſt aperture to have been ſeven feet and a half. Of its length he ſays nothing, and only adds, that it could be heard at the diſtance of 500 ſtadia, or about 25 miles.

However this may be, the ſpeaking trumpet is nothing elſe but a long tube, which at one end is only large enough to receive the mouth, and which goes on increaſing in width to the other extremity, bending ſomewhat outwards. The aperture at the ſmall end muſt be a little flattened to fit the mouth; and it ought to have two lateral projections to cover part of the cheeks.

Sir Thomas Morland ſays, that he cauſed ſeveral inſtruments of this kind to be conſtructed, of different ſizes, viz. one of four feet and a half in length, by which the voice could be heard at the diſtance of 500 geometrical paces; another 16 feet 8 inches, which conveyed ſound 1800 paces; and a third, of 24 feet, which rendered the voice audible at the diſtance of 2500 paces.

The reaſon of this phenomenon is as follows: As the air is an elaſtic fluid, ſo that every ſound produced in it is tranſmitted ſpherically around the ſonorous body, when a perſon ſpeaks at the mouth of the trumpet, all the motion which would be communicated to a ſpherical maſs of air, of four feet radius, for example, is communicated only to a cone, the baſe of which is the wider extremity of the trumpet. Conſequently, if this cone is only the hundredth part of the whole ſphere of the ſame radius, the effect will be the ſame as if the perſon ſhould ſpeak a hundred times as loud in the open air: the voice muſt therefore be heard at a diſtance a hundred times as great.

The ear trumpet, an inſtrument exceedingly uſeful to the deaf, is nearly the reverſe of the ſpeaking trumpet: it collects, in the auditory paſſage, all the ſound contained within it; or it increaſes the ſound produced at its extremity, in a ratio which may be ſaid to be as that of the wide end to the narrow end. Thus, for example, if the wide end be 6 inches in diameter, and the aperture applied to the ear 6 lines, which in ſurfaces gives the ratio of 1 to 144, the ſound will be increaſed 144 times; or nearly ſo; for we do not believe that this increaſe is exactly in the inverſe ratio of the ſurfaces; and it muſt be allowed, that in this reſpect acouſtics are not ſo far advanced as optics.

AMUSING SECRETS.

To make a ring be ſuſpended by a thread after it has been burnt.

NOTHING is neceſſary for this purpoſe, but to employ a thread which has been ſoaked in a ſolution of common ſalt in river water. Though flame be applied to the thread it will ſtill have ſtrength ſufficient to ſuſtain the ring.

To make people in a room have a hideous appearance.

Diſſolve ſalt in an infuſion of ſaffron in ſpirit of wine; then dip ſome tow in the ſolution, and having ſet fire to it, extinguiſh all the other lights in the room.

To form figures in relief on an egg.

Delineate on the ſhell any figures at pleaſure with melted tallow, or other fat oily ſubſtance, proof againſt acids; then immerſe the egg in ſtrong vinegar, and let it remain till the acid has ſufficiently corroded the part of the ſhell not covered with the tallow or oil.

To change a colour from white to blue:

Diſſolve copper filings in a phial of volatile alkali: when the phial is unſtopped, the liquor will be bluiſh; but when ſtopped, it will be white.

To make a red liquor, which, when poured into different glaſſes, ſhall become yellow, blue, black, or purple.

This phenomenon may be produced by the following proceſs: Infuſe a few ſhavings of logwood in common water, and when the liquor is ſufficiently red, pour it into a bottle. Then take three drinking glaſſes, and rinſe one of them with ſtrong vinegar; throw into the ſecond a ſmall quantity of pounded alum, which will not be obſerved if the glaſs has been newly waſhed, and leave the third without any preparation. If the red liquor in the bottle be poured into the firſt glaſs, it will aſſume a ſtraw colour, ſomewhat ſimilar to that of Madeira wine; if into the ſecond, it will paſs gradually from bluiſh gray to black, provided it be ſtirred with a bit of iron, ſuch as a key for example, which has been privately immerſed in good vinegar. In the third glaſs, the red liquor will aſſume a violet tint.

To make pomatum with water and wax, two ſubſtances which do not combine together.

Put into a new glazed earthen pot ſix ounces of river water, and two ounces of wax, in which, to render the proceſs more marvellous, you muſt have concealed a ſtrong doſe of ſalt of tartar. If the whole be then expoſed to a conſiderable degree of heat, it will aſſume the conſiſtence of pomatum, and may be uſed for cleanſing the ſkin.

How a body of a combuſtible nature may be penetrated by fire without being conſumed.

Put into an iron box a piece of charcoal, ſufficient to fill it entirely, and ſolder on the lid. If the box be then thrown into the fire, it will become red, and it may even be left in it for ſeveral hours or days. When opened, after it has cooled, the charcoal will be found entire, though there can be no doubt of its having been penetrated by the matter of the fire, as well as the whole metal of the box which contains it.

Apparent tranſmutation of iron into copper or ſilver.

Diſſolve blue vitriol in water, till the latter is nearly ſaturated, and immerſe into the ſolution ſmall plates of iron or coarſe filings of that metal:

these ſmall plates of iron or filings will be attacked and diſſolved by the acid of the vitriol, while its copper will be precipitated and depoſited in the place of the iron diſſolved. If the bit of iron be too large to be entirely diſſolved, it will be ſo completely covered with cupreous particles, that it will ſeem to be converted into copper. This is an experiment commonly ſhewn to thoſe who viſit copper mines. In Savoy I have ſeen a key become entirely of a copper colour, after being immerſed ſome minutes in water, collected at the bottom of a copper mine.

If you diſſolve mercury in marine acid, and immerſe in it a bit of iron; or if the ſolution be rubbed over the iron, it will aſſume a ſilver colour. Jugglers ſometimes exhibit this chemical deception at the expence of the credulous and ignorant.

REMARK.

In this caſe there is no real tranſmutation, but only the appearance of one. The iron is not changed into copper: the latter, held in ſolution by the liquor impregnated with the vitriolic acid, is only depoſited in the place of the iron with which the acid becomes charged, while it abandons the copper. Every time indeed that a menſtruum, holding any ſubſtance in ſolution, is preſented to another ſubſtance which it can diſſolve with more facility, it abandons the former, and becomes charged with the ſecond. This is ſo certain, that when the liquor which has depoſited the copper is evaporated, it produces cryſtals of green vitriol; which, as is well known, are formed by the combination of the vitriolic acid with iron. This proceſs is indeed practiſed, on a large ſcale, in the mine which I viſited. The liquor in queſtion, which is nothing but a pretty ſtrong ſolution of blue vitriol, is put into caſks, or large ſquare reſervoirs; pieces of old iron being then immerſed in it, are at the end of ſome time converted into a ſort of ſediment, from which copper is extracted. The liquor, thus charged with iron, is evaporated to a certain degree, and wooden rods are immerſed in it, which become covered with cryſtals of green vitriol.

This experiment may be made alſo by diſſolving copper in the vitriolic acid, and then diluting the ſolution with a little water. This is a new proof that the liquor only depoſits the copper with which it is charged.

Different ſubſtances ſucceſſively precipitated by adding another to the ſolution.

In the former experiment we have ſeen copper precipitated by iron: we ſhall now ſhew iron itſelf precipitated. For this purpoſe, throw into a ſolution of iron a ſmall bit of zinc, and in proportion as the latter diſſolves, the iron will fall to the bottom of the veſſel: it may eaſily be known to be iron, becauſe it will be ſuſceptible of being attracted by the magnet.

If you wiſh to precipitate the zinc, nothing will be neceſſary but to throw into the ſolution a bit of calcareous ſtone, ſuch as white marble for example, or any other kind of ſtone capable of making lime: the vitriolic acid will attack this new ſubſtance, and ſuffer to be depoſited at the bottom of the veſſel a white powder, which is zinc.

To precipitate the lime or calcareous earth: pour into the ſolution liquid volatile alkali (ſpirit of hartſhorn); the earth being abandoned by the acid, will depoſit itſelf at the bottom of the veſſel.

The calcareous earth may be precipitated alſo, and much better, by pouring into the liquor a ſolution of fixed alkali, ſuch as fixed vegetable alkali; or by throwing into it fixed mineral alkali.

REMARK.

It is by a ſimilar effect that hard water decompoſes ſoap, inſtead of diſſolving it, and ſuffers to be depoſited a greater or leſs quantity of calcareous earth. The manner in which this is done, is as follows:

Water, in general, is hard only becaufe it holds in folution felenite or gypfum (a combination of vitriolic acid with calcareous earth), which it has diffolved in its paffage through the bowels of the earth, or which has been formed by the water firft becoming impregnated with vitriolic falts, and afterwards in its courfe meeting with, and diffolving, a portion of calcareous earth.

On the other hand, foap is an artificial combination of fixed alkali with oil, or with fome other greafy fubftance, and which have no great affinity.

When foap therefore is diffolved in water impregnated with felenite, the vitriolic acid of the latter having a greater tendency to unite with the fixed alkali, than with the calcareous earth, which enters into the compofition of the felenite, abandons that earth, and combines with the fixed alkali in fuch a manner, that the foap is decompofed; and, as the oil is immiscible with water, it is diffufed through it in the form of white flakes, while the calcareous earth of the felenite falls to the bottom.

By the mixture of two tranfparent liquors, to produce a blackifh liquor: Method of making good ink.

Provide a folution of green or ferruginous vitriol, and an infusion of gall-nuts, or any other astringent vegetable substance, such as oak-leaves, well clarified and filtered; if you then pour the one liquor into the other, the compound will immediately become obscure, and at last black.

If the liquor be suffered to remain at rest, the black matter suspended in it will fall to the bottom, and leave it transparent.

REMARK.

This experiment may serve to explain the formation of common ink; for the ink we use is nothing but a solution of green vitriol mixed with an infusion of gall-nuts, and a little gum. The blackness arises from the property which the gall-nuts have of precipitating, of a black or blue colour, the iron held in solution by the water impregnated with vitriolic acid; but as the iron would soon fall to the bottom, it is retained by the addition of gum which gives to the water sufficient viscosity to prevent the iron from being precipitated.

The reader, perhaps, will not be displeased to find here the following recipe for making good ink.

Take one pound of gall-nuts, six ounces of gum arabic, six ounces of green coperas, and one gallon of common water or beer; pound the gall-nuts, and infuse them in a gentle heat for twenty-four hours, without bringing the mixture to ebullition; then add the gum in powder. When the gum is dissolved, put in the green vitriol. And if you then strain the mixture, you will obtain very fine ink.

To produce inflammable and fulminating vapours.

Put into a moderately sized bottle, with a short wide neck, three ounces of oil or spirit of vitriol, with twelve ounces of common water, and throw into it, at different times, an ounce or two of iron filings. A violent effervescence will then take place, and white vapours will arise from the mixture. If a taper be presented to the mouth of the bottle, these vapours will inflame and produce a violent detonation, which may be repeated several times, as long as the liquor continues to furnish similar vapours.

The philosophical candle.

Provide a bladder, into the orifice of which is inserted a metal tube, some inches in length, that can be adapted to the neck of a bottle, containing the same mixture as that used in the preceding experiment.

Having then suffered the atmospheric air to be expelled from the bottle, by the elastic vapour produced by the solution, apply to the mouth of it the orifice of the bladder, after carefully expressing from it the common air.* The bladder by these means will become filled with the inflammable air; which if you force out against the flame of a taper, by pressing the sides of the bladder, will form a jet of beautiful green flame.—This is what chemists call a philosophical candle.

To make an artificial volcano.

For this curious experiment, which enables us to assign a very probable cause for volcanoes, we are indebted to Lemery.

Mix equal parts of pounded sulphur and iron filings, and having formed the whole into a paste with water, bury a certain quantity of it, forty or fifty pounds for example, at about the depth of a foot below the surface of the earth. In ten or twelve hours after, if the weather be warm, the earth will swell up and burst, and flames will issue out, which will enlarge the aperture, scattering around a yellow and blackish dust.

It is not impossible that what is here seen in miniature, takes place on a grand scale in volca-

* Great care must be taken not to omit this precaution; for a mixture of inflammable and atmospheric air will explode with violence, instead of burning. T.

noes; as it is well known that they always furnifh abundance of fulphur, and that the matters they throw up abound in metallic and probably ferruginous particles; for iron is the only metal which has the property of producing an effervefcence with fulphur, when they are mixed together.

But it may eafily be conceived, from the effect of a fmall quantity of the above mixture, what thoufands or millions of pounds of it would produce: there is no doubt that the refult would be phenomena as terrible as thofe of earthquakes, and of thofe volcanic eruptions with which they are generally accompanied.

To make fulminating powder.

Mix together three parts of nitre, two of well-dried fixed alkali, and one of fulphur: if a little of this mixture be put into an iron fpoon, over a gentle fire, capable however of melting the fulphur; when it acquires a certain degree of heat, it will detonate with a loud noife, like the report of a fmall cannon.

This would not be the cafe if the mixture were expofed to a heat too violent: the parts only moft expofed to the fire would detonate, and by thefe means the effect would be greatly leffened.

If thrown on the fire, it would not detonate, and would produce no other effect than pure nitre, which indeed detonates, but without any explosion.

To form a combination which when cold is liquid and transparent, but when warm becomes thick and opake.

Put equal quantities of fixed alkali, either mineral or vegetable, and of well-pulverised quick lime, into a sufficient quantity of water, and expose it to strong and speedy ebullition; then filtre the product, which at first will pass through with difficulty, but afterwards with more ease, and preserve it in a bottle well stopped. This liquor, when made to boil, either in the bottle or in any other vessel, will become turbid, and assume the consistence of very thick glue; but, when cold, it will recover its fluidity and transparency.

To make a flash, like that of lightning, appear in a room, when any one enters it with a lighted candle.

Dissolve camphor in spirit of wine, and deposit the vessel containing the solution in a very close room, where the spirit of wine must be made to evaporate by speedy and strong ebullition. If any one then enters the room with a lighted candle, the air will inflame, while the combustion will be so fudden, and of fo fhort a duration, as to occafion no danger.

It is not improbable that the fame effect might be produced, by filling the air of an apartment with the duft of the feed of a certain kind of lycoperdon, which is inflammable.

Of Sympathetic Inks, and fome tricks which may be performed by means of them.

Sympathetic inks are certain liquors, which alone, and in their natural ftate, are colourlefs; but which, by being mixed with each other, or by fome particular circumftance, affume a certain colour.

Chemiftry prefents us with a great many liquors of this kind, the moft curious of which we fhall here defcribe.

1ft. If you write with a folution of green vitriol, to which a little acid has been added, the writing will be perfectly colourlefs and invifible. To render it vifible, nothing will be neceffary but to immerfe the paper in an infufion of gall-nuts in water, or to draw a fponge moiftened with the infufion over it.

2d. If you are defirous of having an ink that fhall become blue; you muft write with an acid folution of green vitriol, and moiften the writing with a liquor prepared in the following manner:

Make four ounces of tartar, mixed with the same quantity of nitre, to detonate on charcoal; then put this alkali into a crucible with four ounces of dried ox blood, and cover the crucible with a lid, having in it only one small aperture; calcine the mixture over a moderate fire, till no more smoke issues from it; and then bring the whole to a moderate red heat; take the matter from the crucible, and immerse it, while still red, in two quarts of water, where it will dissolve by ebullition; and when the liquor has been reduced to one half, it will be ready for use. If you then moisten with it the writing, above mentioned, it will immediately assume a beautiful blue colour. In this operation, instead of black ink, there is formed Prussian blue.

3d. If you dissolve bismuth in nitrous acid, and write with the solution, the letters will be invisible. To make them appear, you must employ the following liquor:

Boil a strong solution of fixed alkali with sulphur reduced to very fine powder, until it dissolves as much of it as it can: the result will be a liquor which exhales vapours of a very disagreeable odour, and to which, if the above writing be exposed, it will become black.

4th. Of all the different kinds of sympathetic ink, the most curious is that made with cobalt. It is a very singular phenomenon, that the characters or figures, traced out with this ink, may be made to disappear and re-appear at pleasure: this property is peculiar to ink made with cobalt; for all the other kinds are at first invisible, until some substance has been applied to make them appear: when they have once appeared, they remain.

To prepare this ink, take zaffer, and dissolve it in aqua regia (nitro-muriatic acid) till the acid extracts from it every thing it can; that is to say, the metallic part or the cobalt, which communicates to the zaffer its blue colour; then dilute the solution, which is very acrid, with common water. If you write with this liquor on paper, the characters will be invisible; but when exposed to a sufficient degree of heat they will become green. When the paper has cooled, they will disappear.

It must however be observed, that if the paper be heated too much, they will not disappear at all.

REMARK.

With this kind of ink, some very ingenious and amusing tricks, such as the following, may be performed.

1st. To make a drawing, which shall alternately represent winter and summer.

Draw a landscape, and delineate the ground, and the trunks and branches of the trees, with the usual colours employed for that purpose, but the grass and leaves of the trees with the liquor above mentioned. By these means you will have a drawing, which, at the common temperature of the atmosphere, will represent a winter-piece; but if it be exposed to a proper degree of heat, not too strong, you will see the ground become covered with verdure, and the trees with leaves, so as to present a view in summer.

Screens painted in this manner were formerly made at Paris. Those to whom they were presented, if unacquainted with the artifice, were astonished to find, when they made use of them, that the views they exhibited were totally changed.

2d. The magic oracle.

Write on several sheets of paper, with common ink, a certain number of questions, and below each question write the answer with the above kind of sympathetic ink. The same question must be written on several pieces of paper, but with different answers, that the artifice may be better concealed.

Then provide a box, to which you may give the name of the Sybil's cave, or any other at pleasure, and containing in the lid a plate of iron made very hot, in order that the inside of it may be heated to a certain degree.

Having selected some of the questions, take the bits of paper containing them, and tell the company that you are going to send them to the Sybil, or Oracle, to obtain an answer; introduce them into the heated box, and when they have remained in it some minutes, take them out, and shew the answers which have been written.

You must however soon lay aside the bits of paper; for if they remain long in the hands of those to whom the trick is exhibited, they would see the answers gradually disappear, as the paper becomes cold.

Of Metallic Vegetations.

To see a kind of shrub rise up in a bottle, and even throw out branches, and sometimes a kind of fruit, is one of the most curious spectacles exhibited by chemistry. The operation by which this delusive image is produced, has been called chemical or metallic vegetation, because performed by means of metallic substances; and it is not improbable that some respectable persons, who thought they saw a real palingenesy, have been deceived by a similar artifice. However this may be, the following are the most curious of these vegetations, which in fact are only a sort of crystallizations.

Arbor Martis, or Tree of Mars.

Diſſolve iron filings in ſpirit of nitre (aqua fortis) moderately concentrated, till the acid is ſaturated; then pour gradually into the ſolution a ſolution of fixed alkali, commonly called oil of tartar per deliquium. A ſtrong effervеſcence will take place, and the iron, inſtead of falling to the bottom of the veſſel, will afterwards riſe, ſo as to cover its ſides, forming a multitude of ramifications heaped one upon the other, which will ſometimes paſs over the edge of the veſſel, and extend themſelves on the outſide, with all the appearance of a plant. If any of the liquor is ſpilt, it muſt be carefully collected, and be again put into the veſſel, where it will form new ramifications, which will contribute to increaſe the maſs of the vegetation.

Arbor Dianæ, or Tree of Diana.

This kind of vegetation is called the Tree of Diana, becauſe it is formed by means of ſilver, as the former is called the Tree of Mars becauſe produced by iron.

Mix together two parts of very pure mercury, and four of fine ſilver, in filings or ſcales, by means of trituration with an ivory peſtle in a porphyry mortar; then diſſolve this amalgam in four ounces of very pure ſpirit of nitre, moderately ſtrong, and dilute the ſolution with about a pound and a half of diſtilled water; ſhake the mixture, and preſerve it in a bottle well ſtopped. Pour an ounce of this liquor into a glaſs, and throw into it a ſmall bit, about the ſize of a pea, of an amalgam of mercury and ſilver, ſimilar to the former, and of the conſiſtence of butter. Soon after you will ſee riſing from the ball of amalgam a multitude of ſmall filaments, which will viſibly increaſe in ſize, and, throwing out branches, will form a ſort of ſhrubs.

Non-metallic Vegetation.

Cauſe to decrepitate, on burning charcoal, eight ounces of ſaltpetre, and place it in a cellar, in order that it may produce oil of tartar per deliquium, then gradually pour over it, to complete ſaturation, good ſpirit of vitriol, and evaporate all the moiſture. The reſult will be a white, compact, and very acrid ſaline matter. Put this matter into an earthen diſh, and having poured over it a gallon of cold water, leave it expoſed to the open air. At the end of ſome days the water will evaporate, and there will be formed all around the veſſel ramifications in the form of needles, variouſly interwoven with each other, and about 15 lines in length. When the water is entirely evaporated, if more be added, the vegetation will continue.

It may be readily ſeen that this is nothing but the mere cryſtallization of a neutral ſalt, formed by the vitriolic acid and the alkali of the nitre employed, that is to ſay, vitriolated tartar.

To produce heat, and even flame, by means of two cold liquors.

Put oil of guaiacum into a baſon, and provide ſome ſpirit of nitre, ſo much concentrated, that a ſmall bottle, capable of holding an ounce of water, may contain nearly an ounce and a half of this acid. Make faſt the bottle, containing the acid, to the end of a long ſtick; and, after taking this precaution, pour about two thirds of the acid into the oil in the baſon; the reſult will be a ſtrong effervefcence, which will be followed by a very large flame. If an inflammation does not take place in the courſe of a few ſeconds, you have nothing to do but to pour the remainder of the nitrous acid over the blackeſt part of the oil: a flame will then certainly be produced; and there will remain, after the combuſtion, a very large ſpongy kind of charcoal.

Oil of turpentine, oil of ſaſſafras, and every other kind of eſſential oil, may be made to inflame in the like manner.

The ſame phenomenon may be produced with fat oils, ſuch as olive oil, nut oil, and others extracted by expreſſion, if an acid, formed by equal parts of the vitriolic and nitrous acids, well concentrated, be poured into them.

To fuse iron in a moment, and make it run into drops.

Bring a bar of iron to a white heat, and then apply to it a roll of sulphur: the iron will be immediately fused and run down in drops. It will be most convenient to perform this experiment over a bason of water, in which the drops that fall down will be quenched. On examination, they will be found reduced into a kind of cast iron.

This process is employed for making shot used in hunting: as the drops, by falling in the water, naturally assume a round form.

Cement for mending broken China.

Calcine oyster shells, and having pounded them, sift them through a silk sieve, and grind them on porphyry, till they are reduced to an impalpable powder. Then take the whites of several eggs, according to the quantity of the powder to be used, and form them with the powder into a kind of paste or glue: with this paste join the fragments of the porcelain, and press them together for the space of seven or eight minutes. No longer time is necessary to dry this mastic, which will stand both heat and water, and which will never give way, even if the article, by any accident, should have a fall.

Process for whitening Prints.

Paste a piece of paper to a very smooth table, that the boiling water used in the operation may not acquire a colour, which might lessen its success. When this precaution has been taken, spread out the print on the table, and sprinkle it with boiling water, taking care to moisten it equally throughout, by means of a fine sponge. After this process with boiling water has been repeated three or four times, you will observe the stains or spots extend themselves; but this need excite no uneasiness, as it is only a proof that the dirt imbibed by the paper begins to be dissolved.

After this preparation, the prints must be put into a copper or wooden vessel, of such a size as to admit of their being freely stretched out in it; they are then to be covered with a boiling lye of pot-ash, and care must be taken to keep it hot as long as possible. After the whole has cooled, take out the prints with care; spread them on stretched cords, and when half dry press them between leaves of paper, in order that they may not contract wrinkles.

By this process, spots and stains of every kind may be removed.

Method of taking paintings from the old canvass, and transferring them to new.

Take the painting from its frame, and tack it down on a very ſmooth table, with the face upwards, and in ſuch a manner that it may be well ſtretched, and free from wrinkles; then cover it with a ſtratum of ſtrong glue, and lay over it ſome ſheets of large white paper of the ſtrongeſt kind you can procure. When the whole has dried, draw the tacks, and having inverted the painting, that is, turned the back uppermoſt, without fixing it, dip a ſponge in tepid water, and gradually moiſten the canvass, trying it from time to time at the edges, to ſee whether it begins to detach itſelf from the painting. When you find it ſufficiently looſe, detach it carefully along one of the edges, and fold back the part ſo detached: if you then roll it with both hands, the whole canvass may by theſe means be removed. When this is done, waſh well the back of the painting with a ſponge dipped in water, until all the old ſize has been nearly removed; then cover the back of the painting with a new ſtratum of ſize, or the uſual priming applied to new canvass, intended for pictures, and immediately ſpread over it a piece of new canvass, which muſt be ſomewhat larger than the painting, in order that it may be properly ſtretched and nailed down at the edges. In the laſt place, do over the canvaſs, portion by portion, with a ſtratum of glue, taking care to ſpread it with a painter's muller, ſo that it may paſs through the pores of the cloth to the painting.

When the painting is dry, remove it from the table, and put it into its frame, after which you muſt thoroughly moiſten the paper with a ſponge dipped in warm water, that it may be taken off without leaving any traces behind it, and to waſh out any ſtains that may ſtill remain on the painting. Conclude the proceſs by rubbing over the painting with pure nut oil, and when dry with the white of an egg properly beat up.

To fill a glaſs with water in ſuch a manner, that a perſon ſhall not be able to remove it without ſpilling it all.

Lay a bet with any one that you will fill a glaſs with water, and place it on a table in ſuch a manner, that it cannot be removed without ſpilling the whole water it contains. Then fill a glaſs with water, and placing over it a bit of paper, ſo as to cover the water and the edge of the glaſs; clap the palm of your hand on the paper, and laying hold of the glaſs with the other, ſuddenly invert it on a very ſmooth table. If you then gently draw out the paper, the water will remain ſuſpended in the glaſs, and it will not be poſſible to remove it without ſpilling the water entirely.

To conſtruct two figures, one of which ſhall blow out a candle, and the other immediately light it again.

Prepare two figures of any materials whatever, and insert into the mouth of each a tube of the size of a small quill. Put into one of these tubes a small piece of phosphorus, and into the other a few grains of gun-powder, taking care that each may be retained in the tube by a bit of paper. If the second figure be applied to the flame of a taper, it will extinguish it, and the first applied will light it again.

The same kind of phosphorus may be employed, on the point of a knife, to light a candle which has been newly extinguished.

Japan Vases.

The Japanese have the art of making a kind of vases with the shavings of paper, or with saw-dust, which when covered with varnish are capable of containing hot or cold liquors. These vases, which are exceedingly neat and light, are ornamented in an agreeable manner with flowers, birds and animals, and with gilt borders.

This preparation is called *papier maché*, and is made of the shavings of white or brown paper, boiled in water, and beat in a mortar till they are reduced to a kind of paste. This paste is afterwards boiled with a solution of gum arabic, to give it tenacity, and by being pressed into moulds, rubbed over with oil, it may be formed into toys of various kinds; which when dry are done over with a mixture of glue and lamp-black, and then varnifhed.

The black varnifh ufed for thefe toys, is prepared in the following manner:

Diffolve in a glazed earthen pot, a little colophonium, or boiled turpentine, till it becomes black and friable, and gradually throw into the mixture three times as much amber finely pulverifed; adding from time to time a little fpirit or oil of turpentine. When the amber is diffolved, befprinkle the mixture with the fame quantity of farcocolla gum, continually ftirring the whole, and add fpirit of wine till the compofition becomes fluid: then ftrain it through a piece of hair-cloth, preffing it between two boards. This varnifh, when mixed with ivory black, is applied in a warm place on the dried pafte of the paper fhavings; the articles are then put into a hot ftove; next day they are removed into a hotter ftove; and the third into one ftill hotter: each time they are left till the ftove has cooled. The pafte, when thus varnifhed, is hard, brilliant, and durable, and capable of containing liquors either hot or cold.

To conftruct a veffel, from which water fhall efcape through the bottom, as foon as its mouth is unftopped.

Among the number of the amufing tricks founded on philoſophical principles, we may claſs the following: Provide a veſſel of tin plate, two or three inches in diameter, and five or ſix inches in height, having a mouth about three lines in width and in the bottom ſeveral ſmall holes, of ſuch a ſize as to admit a ſmall needle. Immerſe this veſſel in water, with its mouth open, and when full ſtop it very cloſely. If you are deſirous of playing a trick to any perſon, give him this veſſel, and deſire him to unſtop it; if he does ſo, placing it on his knees, the water will eſcape through the holes in the bottom, ſo that he will ſoon be all over wet.

Tranſparencies.

Thoſe tranſparencies exhibited on the ſtage, and during public feſtivals, which are illuminated by a light placed behind them, are prepared in the following manner. A piece of ſtrong linen or ſilk, ſtretched on a wooden frame, is done over with a ſolution of wax in oil of turpentine, and during the operation a chaffing diſh is placed below it, that the liquid may be every where equally diffuſed. Any figures at pleaſure are then delineated on the cloth with oil colours, mixed up with ſpirit of turpentine.

Moveable tranſparencies, exceedingly amuſing, may be formed in the following manner. Affix the tranſparency to a very light circular frame, ſupported by an axis on which it can freely turn. The upper end of the cylinder must be closed by a circular piece of tin-plate, cut into inclined planes, like the ventilators constructed in windows to prevent smoke: if a lamp be then placed within the cylinder, it will illuminate the transparency, and at the same time make it turn round by the means of the current of air which falls on the tin-plate. The figures exhibited by this transparency may be varied a thousand ways, according to the taste of the artist: they may be made to represent serpents, twisting around a column, &c.

It is by the same mechanism that a spiral piece of card or paper, placed on a stove, turns round of itself, and serves as a thermometer to regulate the heat.

Method of fixing Crayons.

Crayon painting is superior to oil painting in brightness, freshness, splendor of colouring, and fidelity of likeness. It is attended with this advantage also, that it is not subject to that reflection of light which prevents the beauty of a painting from being seen except from a certain point of view. On account of these valuable qualities, it would certainly have been preferred to oil painting, had it been equally durable; but it has this inconvenience, that it is liable to be destroyed by the least friction. At the end of a few years master-pieces of this kind perish, because the powder of the crayons detaches itself, or becomes mouldy, especially if great care be not taken to preserve these paintings from moisture, and from the heat of the sun. The following liquor however has been employed with success for fixing crayons: it is not expensive, and nothing is necessary but to immerse the painting in it for a few moments.

To prepare this liquor, dissolve Roman alum pulverised, in two glass-fulls of very pure water, and when the water is saturated, decant it from off the alum, which may have remained undissolved at the bottom of the vessel. This observation is of great importance; for if the alum which has not been dissolved were left in the liquor, by becoming dry it might tarnish the painting, and produce whitish spots in those places where the liquor accumulates itself in draining off. Into this water, well impregnated with alum, put a small quantity of very transparent and pure fish glue, leaving it to dissolve for twenty-four hours, and then boil the whole, that the glue may be dissolved completely. The liquor must afterwards be strained through a piece of linen, to free it from any impurities it may contain. In the last place, pour the water, thus impregnated with alum and glue, into a bottle containing three pints of brandy not coloured, and mixed with a large glass-full of spirit of wine. A greater or less quantity of this liquor may be made according to the ſize of the paintings to be fixed, provided care be taken to increaſe the ingredients in the proper proportions; it is however to be obſerved, that it muſt not be uſed when too old, as in that caſe it would weaken the ſplendor of the painting.

Put the liquor, thus prepared, into a veſſel of lead, or of any other ſubſtance, ſo large that the painting may be immerſed in it, and heat it in a balneum mariæ, taking care that the fiſh-glue be well diſſolved; for before the liquor is heated, eſpecially if the weather be cold, it will depoſit itſelf at the bottom. Place in each corner of the baſon a bit of lead, in ſuch a manner, that the liquor may riſe over it no more than a line at moſt, and then lay hold of the painting, keeping it in an horizontal poſition, and immerſe it gently into the liquor. The pieces of lead, placed in the veſſel, will prevent it from ſinking too deep. The time employed in immerſing and taking out the painting, ought not to exceed a ſecond.

The painting muſt be taken out horizontally, and be depoſited in the ſame poſition in ſome place where it can reſt on its two borders, which it will do if ſupported by two chairs. If the above proceſs be properly followed, it will be found that all the tints have retained their original freſhneſs and primitive colour. Crayons fixed in this manner will bear even to be covered with a varniſh, which may ſupply the place of glaſs. To lay on this varniſh, the following method is to be employed.

When the painting is fixed and dry, apply over it, with a ſoft bruſh, a ſtratum or two of melted fiſh-glue, mixed with about a third of ſpirit of wine, and ſufficiently ſtrong that when cold it may form a ſort of jelly. When this preparation is dry, apply that varniſh uſed for varniſhing prints, which will produce the ſame effect as on paintings in diſtemper.

Crayon paintings fixed in the above manner are attended with this advantage that they may be retouched; for the crayons will make an impreſſion as before: ſome ſtrengthening touches may even be added with colours in diſtemper. This method employed for crayons, may be uſed alſo for fixing chalk drawings.

A piece of money being put into a baſon, how to make two pieces appear, one of which ſhall be much larger than the other.

Fill a glaſs goblet with pure water, and put into it a piece of money, ſuch for example as a ſhilling; then cover the goblet with a plate, and laying your hand upon the latter, invert the whole ſpeedily, ſo that the air not having time to enter, the water may not be able to eſcape.

If you look at the piece of money, which will then be on the plate, it will appear of the ſize of a half-crown, and it will be ſeen alſo of its real ſize a little above the former image, which will make thoſe unacquainted with the ſingular effects of refraction to believe, that there are really below the goblet a half-crown and a ſhilling. When the goblet is removed, the illuſion will ceaſe.

To make a phantom appear on a pedeſtal placed on a table.

Every body is acquainted with the magic lantern, by means of which, groteſque figures, painted on glaſs, are made to appear on a wall or a white ſheet. The illuſion produced by this machine, may be varied by cauſing the images to be reflected on a column of ſmoke, which will exhibit them very diſtinctly.

By theſe means a phantom may be made to appear ſuddenly, and at pleaſure, above a kind of pedeſtal ſufficiently large to contain a magic lantern towards its bottom, and a chaffing-diſh towards the top, oppoſite to a round aperture made for the purpoſe of affording a paſſage to a current of ſmoke. The image of the ſpectre, by means of a mirror placed at the proper angle, will be reflected on the current of ſmoke, which will repreſent it to the ſpectators.

The figure of the spectre on the lantern-glass must be painted short, because the image on the current of smoke, being projected obliquely, would assume too lengthened a form, which would alter its proportion.

This exhibition will excite great astonishment, because the spectators, being unacquainted with the cause, will be at a loss to account for the sudden apparition of the spectre, the head of which will first appear; and which will seem to rise through the smoke, and to disappear by sinking down into the pedestal. To produce this effect, nothing will be necessary but to pull gently, or to let go, the string, when the smoke has been sufficiently illuminated by the magic lantern.

During this exhibition there must be no lights in the room, and the pedestal must be placed so high, that the spectators cannot see into the inside of it. The aperture through which the cone of light issues, may be kept covered till the moment when the spectre is about to appear. This experiment may be performed on a large scale, so that the phantom shall appear of the size of a man.

An object being placed behind a convex glass, to make it appear before it.

Provide any object, such as for example as a small arrow of white wood, an inch and a half in length, and tie it perpendicularly to a piece of black card, which muſt be ſuſpended from a wall, at about the height of the eye. Throw a ſtrong light on the card, and place before it a lenticular glaſs, two or three inches in diameter, in ſuch a manner, that it may be diſtant from the arrow about twice the length of its focus. If you then make a perſon ſtand at a proper diſtance, oppoſite to the glaſs, the arrow will appear to him to be ſuſpended in the air before the glaſs.

It is evident that this ſingular effect of dioptrics, with taſte and a little ingenuity, may be applied to a variety of other amuſements, which it is needleſs here to detail.

The Chineſe Shadows. Ombres Chinoiſes.

Make an aperture, in a partition wall, of any ſize, for example four feet in length and two in breadth, ſo that the lower edge may be about five feet from the floor, and cover it with white Italian gauze, varniſhed with gum copal. Provide ſeveral frames of the ſame ſize as the aperture, covered with the ſame kind of gauze, and delineate upon the gauze different figures, ſuch as landſcapes and buildings, analogous to the ſcenes which you intend to exhibit by means of ſmall figures repreſenting men and animals.

These figures are formed of paſteboard, and their different parts are made moveable according to the effect intended to be produced by their shadows, when moved backwards and forwards behind the frames, and at a small distance from them. To make them act with more facility, small wires fixed to their moveable parts, are bent backwards, and made to terminate in rings, through which the fingers of the right hand are put, while the figure is supported by the left, by means of another iron wire: in this manner they may be made to advance or recede, and to gesticulate, without the spectators observing the mechanism by which they are moved; and as the shadow of these figures is not observed on the paintings till they are opposite those parts which are not strongly shaded, they may thus be concealed, and made to appear at the proper moments, and others may be occasionally substituted in their stead.

It is necessary, when the figures are made to act, to keep up a sort of dialogue, suited to their gestures, and even to imitate the noise occasioned by different circumstances: The paintings must be illuminated from behind, by means of a reverberating lamp, placed opposite to the centre of the painting, and distant from it about four or five feet.

Various amusing scenes may be represented in this manner, by employing small figures of men and animals, and making them move in as natural a way as possible, which will depend on the address and practice of the person who exhibits them.

To direct a swarm of Bees at pleasure.

It is well known that the female bee is the queen of the hive, and that the fate of the whole swarm depends, in some measure, upon her alone. The distinguishing characters of this mother bee are, that she has very short wings: it is difficult for her to fly, and therefore she seldom goes abroad, except when she quits the hive to form a new colony. On that occasion, the bees, like faithful subjects, follow her to whatever place she may have chosen, and for this reason, if a person can get possession of the queen bee, he is sure of being able to direct the swarm at his pleasure. In that case, nothing is necessary but to confine her by means of a hair, or a very fine thread of silk, made gently fast around her corslet; the bees, attentive to all her actions, will surround her, go backwards and forwards, stop and seem obedient to the will of him who commands the mother bee, by merely following the movements of their queen.

This was the charm, or rather the secret, by which Mr. Wildman, who had studied the instinct of bees, and who thus took advantage of their attachment for their queen, was able to make a swarm pass from one hive to another at pleasure. Having full confidence in the success of his experiments, he presented himself one day to the Society of arts, with three swarms of bees which he brought along with him, partly on his face and shoulders, and partly in his pockets. He placed the hives to which these swarms belonged in an outer apartment, and on blowing a whistle they all immediately quitted him, and returned to their hives; but on blowing his whistle a second time, they returned to occupy their former place on the person, and in the pockets of their master. This exercise was repeated several times, to the great astonishment of the society, without any of the spectators being injured.

These astonishing experiments, the secret cause of which we have explained, were repeated some years ago, with equal success, before the Academy of Sciences at Paris, by Mr. Wildman, who explained to the French Academicians the theory and practice of his wonderful art.

A powder which inflames when exposed to the air.

Put three ounces of rock alum and one ounce of honey, or sugar, into a new, glazed earthen dish, capable of standing a strong heat, and keep the mixture over the fire, stirring it continually, till it become very dry and hard: then remove it from the fire, and pound it until it assume the form of a coarse powder.

Put this powder into a small matrass, or long-necked bottle, leaving part of the vessel empty, and having placed it in a crucible, fill up the crucible with fine sand, and surround it with burning coals.

When the matrafs has been kept at a red heat for about feven or eight minutes, and no more vapour iffues from it, remove it from the fire; then ftop it with a piece of cork; and, having fuffered it to cool, preferve the mixture in fmall bottles well clofed.

If you uncork one of thefe bottles, and let fall on a bit of paper, or any other very dry fubftance, a few grains of this powder, it will firft become bluifh, then brown, and will be speedily converted into an ardent body, fo as to burn the paper or any other combuftible fubftance on which it may have been expofed.

When a few grains of this powder catch fire, on being thus expofed to the air, they emit a light flame, which refembles that of common fulphur when it begins to burn; and they exhale, at the fame time, an odour fimilar to that produced by the fmoke of fulphur.

Fulminating Gold.

Put into a fmall matrafs, refting on a little fand, one part of fine gold-filings and three parts of aqua-regia (nitro-muriatic acid). When the filings are completely diffolved, pour the folution into a glafs, and add to it five or fix times the quantity of common water.

Then take fpirit of fal ammoniac, or oil of tartar, and pour it drop by drop into this folution, until the gold is entirely precepitated to the bottom of the glaſs; decant the ſupernatant liquor, by inclining the glaſs, and having waſhed it ſeveral times in tepid water, dry it at a very moderate heat, placing it on paper capable of abſorbing all the humidity.

If a grain of this powder, put into a metal ſpoon, be expoſed to the flame of a taper, as ſoon as it becomes ſufficiently heated, it will explode with a very loud report; but it ſometimes happens that it pierces the ſpoon and forces itſelf downwards with great violence.

Method of Cutting Glaſs by the means of Heat.

Take a common drinking glaſs, not very thick, and apply to the edge of it a lighted match, until the violence of the heat produces a crack in it; then move the match along the crack, following a ſpiral direction, and after five or ſix circum-volutions, the glaſs will form a ſort of ſcroll, the parts of which will ſeparate when you invert it; but which will be re-joined when put again into its natural poſition.

This method may be employed to cut glaſs tubes; for if a ſmall notch be made with a file in the place where the tube is to be divided, you may eaſily make it ſplit in that place, by applying to it a piece of angular iron made red hot.

To fuse a piece of money in a walnut ſhell, without injuring the ſhell.

Bend any very thin coin, and having put it into the half of a walnut ſhell, place the ſhell on a little ſand, in order that it may remain ſteady. Then fill the ſhell with a mixture made of three parts of very dry pounded nitre, one part of the flowers of ſulphur, and a little ſaw-duſt well ſifted.

If you then inflame the mixture, as ſoon as it has melted you will ſee the metal completely fuſed in the bottom of the ſhell, under the form of a button, which will become hard when the burning matter around it is conſumed: The ſhell employed for the operation will have ſuſtained very little injury.

A ſmall ball of lead cloſely wrapped up in a bit of paper* may be fuſed in the ſame manner, by expoſing it to the flame of a candle. The paper will not be hurt, except in the bottom, where it will have a ſmall hole, through which the metal has run.

Phoſphorus.

The name of phoſphorus is given to certain bodies which ſhine or appear luminous in the dark. Some kinds of it are natural, and others artificial:

* If the paper be not in perfect contact with the lead the experiment will not succeed. T.

the natural are those which shine without the assistance of art, such as certain kinds of rotten wood, glow worms, and almost all fish when they begin to become putrid.

The artificial kinds of phosphorus are those prepared by art; such as Kunckel's phosphorus *, the sulphuret or the sulphate of barytes calcined, called Bologna phosphorus, &c.

A Liquor which Shines in the Dark.

Take a bit of Kunckel's phosphorus, about the size of a pea, and having divided it into several portions, put them into half a glass-full of very pure water, and boil it in a small earthen vessel, over a very moderate fire. Have in readiness a long narrow bottle, with a well fitted glass stopper, and immerse it, with its mouth open, into boiling water. On taking it out, empty it of the water, and immediately pour into it the mixture, in a state of ebullition; then put in the stopper, and cover it with mastic, to prevent the external air from entering it.

This water will shine in the dark for several months, even without being touched; and if it be shaken during dry, warm weather, a kind of brilliant flashes will be seen to rise through the middle of the water.

* Kunckel's phosphorus is the common phosphorus of the shops. T.

Various amuſing tricks may be performed with this phoſphorus, by covering the bottle which contains it with black paper, having words or figures cut out in it: as you may not only cauſe different words to appear, but may even conceal with one of your fingers ſome of the letters, which compoſe them, ſo as to form other words, it will ſeem as if you had the power of making them appear at pleaſure.

To make Luminous Characters appear on a piece of paper, or a wall, &c.

If any characters be traced out with a ſmall bit of Kunckel's phoſphorus, they will appear luminous in the dark. If this experiment be made during warm weather, the light will be more vivid, and will be the ſooner diſſipated, than if performed during cold or moiſt weather: by breathing on theſe characters they will diſappear; but a moment after they will re-appear of themſelves.

A Liquor ſhut up in a Bottle, which when the bottle is unſtopped, becomes Luminous.

Put a little of Kunckel's phoſphorus into eſſence of cloves and fill with it a bottle, which muſt be kept cloſely ſhut: every time the bottle is unſtopped, the whole liquor will appear luminous. This experiment, as well as all the preceding, muſt be performed in the dark.

Kunckel's phoſphorus may be preſerved in a bottle filled with water; but it muſt be put back into the bottle as ſoon as it has been uſed, and care muſt be taken not to touch it with the naked fingers, becauſe it would burn them, and occaſion very acute pain *.

Artificial Memory.

You may lay a bet with any perſon, that you will dictate a great many different numbers at hazard, which he may write down and keep by him, and that you will recite them in the ſame order immediately, or at the end of a month or even of ſeveral years. For this purpoſe it will be neceſſary to be familiariſed with the following numerical alphabet.

A	B	C	D	E	F	G	H
1	2	3	4	5	6	7	8
I	K	L	M	N	O	P	Q
9	10	11	12	13	14	15	16
R	S	T	V	X	Y	Z	&
17	18	19	20	21	22	23	24

As every body knows by heart ſome lines of poetry, or ſome prayer, the whole myſtery of this artifice conſiſts in dictating the figures correſponding to each of the letters of the words employed; obſerving, in order that the numbers may be greatly varied, to join the letters in each word two and two, and to take only the laſt when the number of them is odd.

* It is impossible to be too careful in handling this dangerous substance. T.

For example if the following words be used:

Pallida mors æquo pede pulsat.

You must dictate 151; 1111; 94; 1; 1214; 1718; 516; 2014; 155; 45; 1520; 1118; 119. The truth of this is proved by what follows:

pa — *ll* — *id* — *a* — *mo* — *rs* — *æq* — *uo* —
15,1 11,11 9,4 1 12,14 17,18 5,16 20,14
pe — *de* — *pu* — *ls* — *at*
15,5 4,5 15,20 11,18 1,19.

This recreation will excite great astonishment in those not possessed of the key; who will believe that the numbers have been dictated at hazard, and that they could not be recited at the end of a certain time, without a prodigious memory.

To cause a person to believe that you can make appear to another, shut up in a room by himself, any thing that the former chooses.

This amusement must be performed in concert with some person, in the company, to whom you have communicated the secret.

Agree privately with the person who is to be shut up in the room, that when he hears you give one knock, it denotes the letter A; that two will denote the letter B, and so on according to the order of the 24 letters of the alphabet. Then say, that you will cause appear to the person who will consent to shut himself up in the next room, whatever animal any other in the company chooses; and in order that no other perſon, except he who is in the ſecret, may offer to ſhut himſelf up, tell the company that it will require great courage. The perſon with whom you are in concert muſt then offer himſelf, and you muſt ſeem to accept of him with reluctance. Then kindle a lamp that emits an obſcure light, and having given it to the perſon, deſire him to place it in the middle of the room, and not to be frightened at what he ſees.

When the perſon is ſhut up, take a ſquare piece of black paper, with a bit of white chalk, and deſire any one of the company to write on it the name of the animal which he wiſhes to appear to the perſon in the adjoining room. We ſhall ſuppoſe that he writes the word *cock*. When you have read what has been written, burn the paper, by applying it to the flame of a lamp; and putting the aſhes into a mortar, throw over them ſome powder, which you muſt pretend to be poſſeſſed of great virtue. Then take a peſtle, as if intending to triturate the mixture, and give three knocks with it to denote to the perſon ſhut up in the next room the letter C; after which you muſt move the peſtle round in the mortar two or three times, that the perſon may have leaſure to pay proper attention. Then give fourteen knocks to denote the letter O; and continue in this manner giving the proper knocks for the other two letters. If you then aſk the perſon what he ſees, he will at firſt return no anſwer, in order to make it be believed that he is frightened; but he will at length ſay that he thought he ſaw a cock.

That there may be no miſtake in regard to the letters, each of the parties in the ſecret, before the knocks for the different letters are given, may repeat privately the letters of the alphabet according to their order, ſo as to be certain of the number of the knocks proper for each.

Method of making Silhouettes, or ſhaded Portraits in Miniature.

The method of making ſilhouettes after nature, and as large as life, is exceedingly ſimple. Nothing is neceſſary for this purpoſe, but to place the perſon's head at a proper diſtance, between a candle and a ſheet of paper, and then to trace out the profile correctly with a pencil.

The method of reducing theſe portraits to the miniature ſize depends on the principles of the camera obſcura.

Baptiſta Porta, a philoſopher of the ſixteenth century, obſerved that the external objects appeared like ſhadows painted in the walls, and on the ceiling of his apartment. Nothing eſcapes the eyes of an enlightened observer; every thing, thoſe even which ſome might conſider as of little importance, ſeem worthy of his attention; and the moſt noble diſcoveries are often the reſult of this happy ſagacity. Porta was agreeably ſurprized at the above ſingular effect, and to improve it, he conceived the idea of fixing in the hole of his window a ſmall lens; ſuch was the origin of the camera obſcura. This machine is now rendered portable; but, in whatever manner conſtructed, it is ſtill founded on the ſame principles.

It conſiſts of a lens or object-glaſs, which conveys the rays of light, proceeding from external objects, to any white ground, at a proper diſtance in a darkened room, ſo as to make them repreſent theſe objects in their proper colours, but in an inverted poſition. To cauſe them to appear in their natural poſition, a reflecting mirror is for the moſt part uſed; but the mechaniſm is exceedingly ſimple, and any ingenious perſon, who poſſeſſes the leaſt knowledge of catoptrics or dioptrics, may conſtruct a camera obſcura of any form at pleaſure.

That employed for making miniature ſilhouettes, is nothing else than a box conſtructed of paſteboard or wood, and having in one ſide of it a ſmall hole.

When this hole is turned towards objects, ſtrongly illuminated by the light of the ſun, or of a candle, theſe objects will appear painted in their natural colours on the oppoſite ſide of the box.

If, inſtead of a ſmall hole, there be made one of two or three inches in diameter, with a lens fitted into it, the objects will appear painted in much ſtronger colours; but if a mirror be fixed in the box, at a proper diſtance from the lens, and inclined at an angle of 45 degrees, the objects will be painted in miniature on the upper part of the box made of glaſs; and of greater or leſs ſize, according as the lens is nearer to, or farther from, the mirror.

Silhouettes made as large as nature, by the proceſs above indicated, can be reduced to a very ſmall ſize, by being placed at a proper diſtance from the lens; but if the original be placed at that diſtance, inſtead of the ſilhouette, you will have the pleaſure of ſeeing painted on the glaſs certain parts not expreſſed in a common ſilhouette, ſuch as the eyes, ears, hair, &c.

Method of ſpeedily delineating all ſorts of plants and flowers.

Provide two balls, and ſome printer's ink; then, holding one of the balls in the left hand, place upon it the leaf or plant, the impreſſion of which you are deſirous of obtaining, and taking the other ball, which muſt be daubed over with ink, in the right hand, ſtrike it gently once or twice againſt the plant, without deranging it: Then carefully remove the leaf or plant, and putting it between a ſheet of paper folded double, lay it on a table covered with a woolen cloth, and preſs it two or three times with a wooden roller, covered with a handkerchief, or any thing elſe of the like kind. After this proceſs, you will find on each leaf of the paper, an impreſſion of the upper and lower ſide of the leaf; which, beſides being a perfect reſemblance of nature, will even ſurpaſs the moſt beautiful engravings, eſpecially if the operation has been performed with dexterity.

The Changeable Roſe.

Take a common full-blown red roſe, and having thrown a little ſulphur finely pounded into a chaffing-diſh with coals, expoſe the roſe to the vapour. By this proceſs the roſe will become whitiſh; but if it be afterwards immerſed ſometime in water, it will reſume its former colour.

The Magic Picture.

Provide a glaſs ſimilar to thoſe uſed for miniature paintings, that is to ſay, ſomewhat concave, and another piece of common glaſs of the ſame ſize, and exceedingly thin. Fill the concave ſide of the former with a mixture of hog's lard and wax melted together; then apply the two pieces of glaſs to each other exactly, that the above compoſition may be incloſed between them; and, having wiped the edges very clean, cement upon them, with fiſh glue, a ſmall ſlip of ſwines's bladder. When it is thoroughly dry, clean the glaſſes and apply to the flat ſide a portrait, or any other ſubject at pleaſure, and incloſe the whole in a frame, ſo as to conceal the edges.

If this portrait be expofed to heat, the compofition between the two glaffes will diffolve, and become trafparent, and the portrait will be diftinctly feen; but it will difappear when the fubftance cools: in this manner it may be made to re-appear as often as you choofe.

The Changeable Picture.

Paint upon thin paper, in a flight manner, and with very light colours, any fubject at pleafure, but difpofed in fuch a manner that, by painting the paper ftronger on the other fide, it may be entirely difguifed: Then cover the laft fide with a piece of white paper, to conceal the fecond fubject, and inclofe the whole in a frame, and even between two pieces of glafs.

If you hold this picture between you and the light, and look through it, a fubject will be feen very different from that which it exhibits when looked at in the ufual manner.

Golden Ink.

As writing, before the invention of printing, was the only method of tranfmitting to pofterity the works and difcoveries of celebrated men, it became in the fourteenth and fifteenth centuries an art much cultivated, and in which many perfons excelled. The manufcripts of thofe periods contain writing, the neatneſs and regularity of which are aſtoniſhing. Tranſcribers were even acquainted with a method of ornamenting the initial letters with gold, which they applied in ſuch a manner as to preſerve all its ſplendour. Writing, by the invention of printing, having become of leſs importance, ſoon degenerated, and the ſecret of applying gold to paper and parchment, like many other arts, was at length loſt. The Benedictines however re-diſcovered this ſecret, and ſpecimens of the proceſs, and parchment contaning writing in gold letters, as brilliant as thoſe ſo much admired in the ancient manuſcripts, have been ſeen at the Abbey Saint-Germain-des-Pres at Paris. This proceſs may be exceedingly uſeful, and may furniſh hints for improving ſome of the other arts, which are all connected, and mutually tend to promote each other.

The following is the proceſs, tranſlated from the German.

Take a certain quantity of gum arabic, the whiteſt is the beſt; and, having reduced it to an impalpable powder in a braſs mortar, diſſolve it in ſtrong brandy and add to it a little common water to render it more liquid. Provide ſome gold in a ſhell, which muſt be detached, in order to reduce it to a powder. When this is done, moiſten it with the gummy ſolution, and ſtir the whole with your finger, or with a ſmall hair-bruſh; then leave it at reſt for a night, that the gold may be better diſſolved. If the compoſition becomes dry during the night, it muſt be diluted with more gum water, in which a little ſaffron has been infuſed; but care muſt be taken that the gold ſolution be ſufficiently liquid to be employed with the pen. When the writing is dry, poliſh it with a dog's tooth.

Another proceſs.

Reduce gum ammoniac to powder, and diſſolve it in water in which gum arabic has been previouſly diſſolved, and to which a little garlic juice has been added. This water will not diſſolve the gum ſo as to form a tranſparent fluid; for the reſult will be a milky liquor. With this liquor you muſt form your letters, or ornaments, on paper or vellum, by means of a pen or hair-bruſh: then ſuffer them to dry, and afterwards breathe on them ſome time, till they become ſomewhat moiſt, and immediately apply a few bits of gold leaf cut to the ſize of the letters; preſs the gold leaf gently with a ball of cotton, or bit of ſoft leather, and when the whole is dry, take a ſoft bruſh and draw it gently over the letters, to remove the ſuperfluous gilding. The parts which you wiſh to poliſh, and render brilliant, may then be burniſhed with a dog's tooth.

White Ink, for writing on Black Paper.

Take egg-ſhells, and having carefully waſhed them, remove the internal pellicle, and grind them on a piece of porphyry. Then put the powder into a ſmall veſſel filled with pure water, and when it has ſettled at the bottom, decant the water, and dry the powder in the ſun. This powder muſt be preſerved in a bottle. When you are deſirous of uſing it, put a ſmall quantity of very pure gum ammoniac into diſtilled vinegar, and leave it to diſſolve during the night; next morning the ſolution will appear exceedingly white, and if you then ſtrain it through a piece of linnen cloth, and add to it the powder of egg-ſhells, in ſufficient quantity, you will obtain a very white ink.

Red Ink.

Boil four ounces of Brazil wood in two pints of water, for a quarter of an hour, and having added a little alum, gum arabic, and ſugar-candy, ſuffer the whole to boil for a quarter of an hour longer. This ink may be preſerved a long time, and the older it grows, it will ſtill become redder.

Blue Ink.

Blue ink may be obtained by diluting indigo and ceruſe in gum water.

Yellow Ink.

Take ſaffron and yellow berries, *(graine d'Avignon)*, or gamboge, and dilute them, as before, in gum water.

Green Ink.

This ink is made by boiling ſap-green in water, in which a little rock alum has been diſſolved.

Ink of different colours made from the juice of violets.

Dip a camel's hair bruſh in any acid, ſuch as diluted ſpirit of vitriol, and draw it over a part of the paper. When the liquor is dry, write on it with a pen dipped in violet juice, and the writing will immediately appear of a beautiful red colour.

If a camel's hair bruſh, dipped in an alkaline ſolution, ſuch as that of ſalt of wormwood in water, be drawn over the other part of the paper, by writing on it when dry with juice of violets, you will obtain characters of a beautiful green colour.

If you write with the juice of violets, and draw a bruſh dipped in ſpirit of vitriol over one part of the writing, and a ſecond dipped in ſpirit of hartſhorn, or a ſolution of ſalt of wormwood diſſolved in water, over another, you will have red and green writing.

By exposing this writing to the fire, it will become yellow.

If you write on paper with an acid, such as lemon-juice, (which is as proper for this purpose as any other) and then suffer it to dry, the writing will be invisible till brought near the fire, when it will become as black as ink: The juice of onions produces the same effect.

The older writing of this kind is, the more beautiful the colour becomes; and in like manner the longer the spirit of vitriol, or solution of salt of wormwood, &c. has been left to dissolve, before they are used to write with, the brighter will be the colours.

Tracing Ink.

This name is given to a kind of ink employed for tracing out figures, and other subjects, intended to be engraved, as by means of pressure it may be transferred from paper, and fixed on the white wax with which engravers cover their plates.

To compose this ink, take gunpowder finely pounded, and add to it an equal quantity of printer's black; then put the whole into water with a little Roman vitriol, and stir the mixture, giving it such a consistence that it may be neither two thin nor too thick. Before the ink is used, shake it well, because the black is apt to deposit itself at the bottom of the vessel.

China or Indian Ink.

China ink, which is employed for ſmall drawings and plans, may eaſily be made by the following proceſs. Take the kernels of the ſtones of apricots, and burn them in ſuch a manner as to reduce them to powder, but without producing flame; which may be done by wrapping up a ſmall packet of them in a cabbage leaf, and tying round it a bit of iron wire. Put this packet into an oven, heated to the ſame degree as that required for baking bread, and the kernels will be reduced to a ſort of charcoal, with which an ink may be made ſimilar to that brought from China.

Pound this charcoal in a mortar and reduce it to an impalpable powder, which muſt be ſifted through a fine ſieve; then form a pretty thick ſolution of gum arabic in water, and, having mixed it with the powder, grind the whole on a ſtone, in the ſame manner as colour-men grind their colours. Nothing is then neceſſary, but to put the paſte into ſome ſmall moulds, formed of cards, and rubbed over with white wax, to prevent it from adhering to them.

In regard to the ſmell of the China ink, it ariſes from a little muſk, which the Chineſe add to the gum water, and may eaſily be imitated, The figures ſeen on the ſticks of China ink, are the particular marks of the manufacturers, who, as in all other countries, are desirous of distinguishing whatever comes from their hands.

Dr. Lewis thinks, from the information of Father du Halde, that China ink is composed of nothing but lamp-black and animal glue. Having boiled a stick of China ink in several portions of water, in order to extract all the soluble parts, and having filtered the different liquors, which he evaporated in a stone vessel, he found that the liquors had the same odour as glue, and that they left, after evaporation, a pretty considerable quantity of a tenaceous substance, which seemed to differ in nothing from common glue.

Ink Powder.

Common liquid ink, the method of making which we have already described, is not easily transported from one place to another; and, besides this inconvenience, it is apt to dry in the ink-holder. In bottles, unless well corked, it becomes decomposed and evaporates; and if the bottles happen to break, it may spoil clothes, or any other articles near it. For the convenience therefore of those who travel either by land or by sea, ink powder has been invented, which is nothing else than the substances, employed in the composition of common ink, pounded and pulverised; so that it can be converted into ink in a moment, by mixing it up with a little water.

Method of reviving old writing.

It is often neceſſary to conſult old charters, titles, deeds, and manuſcripts, written many centuries ago, either to gratify curioſity, or to clear up ſome important point in law; but as the writing is ſometimes ſo much effaced as to be ſcarcely legible, a Benedictine invented a liquor, which will make old manuſcripts appear as freſh as if newly written. The proceſs for preparing this liquor, which may be eaſily applied, is as follows:

Having provided a pot, capable of containing three quarts of water, take ſome white onions, freed from the exterior thick ſkin, and cut them into ſmall morſels; put ſuch a quantity of them into the pot as to occupy three fourths of it; then fill up the remaining part with water, and add three gall-nuts well pounded. Boil the whole for an hour and a half, and throw into the mixture about the ſize of a nut of rock alum. Strain the mixture through a piece of cloth, ſqueezing the onions ſtrongly to expreſs the juice, and preſerve the liquor, which, when cold, will have the appearance of orgeat.

When you intend to uſe this liquor, expoſe it to heat, which will render it clear; then dip in it a bit of rag, and apply it to the writing which you wiſh to revive; if you then hold the writing near the fire, that the liquor may make a ſtronger impreſſion, you will have the pleaſure of ſeeing the characters revived in their full luſtre. If there be only a few words of the writing effaced, it will be ſufficient to heat a little of the liquor in a ſilver ſpoon, and to apply it as above.

Another proceſs, ſtill ſimpler, conſiſts in putting three or four pounded gall-nuts into a certain quantity of ſpirit of wine; heating the mixture and expoſing to the vapour of it the writing which you wiſh to revive.

Old papers or parchments, the writing of which cannot be read, or can be read only with difficulty, may be immerſed alſo in water in which copperas has been diſſolved: if they are then ſuffered to dry, the copperas will make the writing re-appear with as much freſhneſs as if it were new.

Method of taking off the Impreſſion of any Drawing.

The impreſſion of any drawing may be taken off, by placing a piece of glaſs over the original, and then tracing out all the outlines with a bit of ſoft red chalk; but as red chalk makes no mark upon glaſs, it muſt firſt be rubbed over with gum water, to which a little vinegar has been added: when the gum is dry it will be fit for drawing on. Without vinegar, red chalk would not mark on the gum; but if you rub the glaſs with the white of an egg, inſtead of gum, there will be no need of vinegar. When the drawing has been traced out on the glaſs, if you apply to it a piece of moiſtened paper, preſſing it ſtrongly down, and immediately remove it, leſt it ſhould adhere to the glaſs, you will find imprinted on it the drawing made with the red chalk. By theſe means, you may obtain an exact outline of any drawing, or print, which you wiſh to copy. This reſemblance, however, will be reverſed; and for that reaſon, to give it the ſame appearance as the original, it muſt be re-copied.

To take off the impreſſion of Old Prints.

Take Venice or Windſor ſoap, which muſt be cut into ſmall morſels, a certain quantity of potaſh, with as much quick lime, and boil the whole in a pot. Wet the engraved ſide of the print gently with this liquor; then apply to it a ſheet of white paper, and roll it ſeveral times with a roller, in order that the impreſſion may be complete.

Method of Teaching Drawing to Young Perſons.

An artiſt propoſes to teach young perſons the elements of drawing, by making them firſt practice on a ſlate; becauſe it may be ſoon cleaned with a wet cloth, or ſponge. This method indeed would ſave the expence of paper, and afford the pupils an opportunity of eaſily correcting their faults, without being obliged to begin their drawing again entirely. For my part, I think it would be more advantageous to employ, inſtead of a ſlate a piece of Bohemian glaſs, which might be made rough on one ſide, by rubbing it with a pumice ſtone, or a flat bit of free ſtone, or fine ſand well moiſtened. Whatever figures have been drawn on this glaſs, may be effaced by a wet cloth, in the ſame manner as from a ſlate; and beſides this advantage, as the glaſs is tranſparent, correct copies may be placed below it, which the ſcholars ought to follow till their hand is properly formed. What is here ſaid of drawing, may be applied alſo to writing.

Method of conſtructing a Lantern, which will enable a perſon to read by night at a very great diſtance.

Make a lantern of a cylindric form, or ſhaped like a ſmall caſk placed lengthwiſe, ſo that its axis may be horizontal, and fix in one end of it a parabolic or ſpheric mirror, ſo that its focus may fall about the middle of the axis of the cylinder: if a ſmall lamp or taper be placed in this focus, the light, paſſing through the other end, will be reflected to a great diſtance, and will be ſo bright, that very ſmall letters on a remote object may be read, by looking at them with a good teleſcope. Thoſe who ſee this light, if they be in the direction of the axis of the lantern, will think they ſee a large fire.

Method of leſſening the danger which ariſes from the agitation of the water, either in the open ſea or on rivers.

Mr. Achard, of the Academy of Sciences of Berlin, in conſequence of ſeveral experiments which he made, was induced to publiſh the following method.

Provide ſeveral caſks, filled with air, and made ſo cloſe as to prevent the entrance of water; or, what will be ſtill better, boxes of tin plate, ſix or eight feet ſquare, and two feet in height, filled alſo with air, and rendered completely water-tight. Ships, without much incumbrance, may always carry with them ſome dozens of theſe caſks, or boxes, made faſt to ropes, and nothing will be neceſſary but to throw them into the water, when it becomes ſo agitated as to give reaſon for apprehending danger; experiments made on a ſmall ſcale have proved, that this method will completely anſwer the intended purpoſe.

Having three veſſels, one capable of containing 8 *pints and full of liquor, and other two empty ones, one capable of containing* 3 *and the other* 5 *pints, to divide the* 8 *pints into two equal portions.*

	P.	P.	P.
Let the three veſſels be - - -	8	5	3
Fill the three-pint veſſel - -	5	0	3
Pour theſe three pints into the one capable of containing five - -	5	3	0
Fill the three-pint veſſel a ſecond time	2	3	3
Pour two pints, from the laſt veſſel, into the five-pint one - -	2	5	1
Pour back the five pints into the eight-pint veſſel - - -	7	0	1
Pour what remains in the three-pint veſſel into the five-pint one - -	7	1	0
Then take three pints from the eight-pint one - - -	4	1	3
Put theſe three pints into the five-pint veſſel - - -	4	4	0

By theſe different tranſpoſitions, the eight-pint veſſel and that of five pints will be found to contain each four pints.

Another method of ſolving this problem.

	P.	P.	P.
Fill the five-pint veſſel - -	3	5	0
Take from theſe five pints as many as will fill the three-pint veſſel -	3	2	3
Put theſe three pints into the eight-pint veſſel - - -	6	2	0

Put into the three-pint veſſel the two pints

which remain in the five-pint one	6	0	2
Fill once more the five-pint veſſel	1	5	2
Fill up the three-pint veſſel by taking one pint from the five-pint one	1	4	3
Pour theſe three pints into the eight-pint veſſel	4	4	0

The reſult is here the ſame as that of the laſt operation.

Conductors.

As places expoſed to lightning ought to be provided with the means of ſecuring them from danger, an apparatus has been invented for this purpoſe, which conſiſts of a rod of iron, terminating in a ſharp point, raiſed above the top of the edifice, and conveyed down to the earth.

This apparatus is called a conductor; and long experience has ſo fully juſtified this appellation, that it is now erected on veſſels, and even on powder magazines, which are often in danger of being blown up by lightning.

In our opinion, a conductor ought to be an inch ſquare, and to riſe ten or twelve feet above the edifice; the point of the ſummit ought to be well gilded, and all the reſt ſhould be varniſhed to prevent it from ruſting, that it may attract the electric fire with more ſtrength, and tranſmit it to the earth which is the common reſervoir.

A certain philosopher has proposed a portable conductor, to be put together and taken to pieces in an instant; and which, in a moment, could be converted from an umbrella into a conductor, or from a conductor into an umbrella.

Were it only necessary to discharge a very strong electric battery, such means might be adequate to the intended purpose; but the effects produced by the most powerful electric machine, can never be compared with the agency of natural electricity. Have we not frequent examples of iron rods being fused by lightning; and is it not to be apprehended that the electric current, directed by the point of this pretended conductor, might fall upon the person who carries it and destroy him? There have been martyrs to electricity, as well as to ærostation.

Cork Jackets, for supporting people in the water.

The greater part of the arts, which ought to be subservient to the use of man, are indebted for their origin merely to avarice and other passions, which rule with despotic sway. Cookery, reduced to systematic rules; factitious liquors, and the frivolous caprice of fashion, seem to exhaust all their resources, to shorten our lives and to destroy our fortunes. We adopt the costume of a Brutus and a Phryne, and then laugh at the shameful depravation of manners which prevails. He who, by the reſult of learned and laborious reſearches, ſhould be able to free us from a number of ſcourges which ſweep off whole generations, would not perhaps meet with ſo much encouragement for his valuable diſcoveries, as the inventor of ſome of thoſe frivolous ornaments, which are as changeable as the phaſes of the moon.

We have long heard of the means invented to ſecure people from the fatal effects of ſhipwreck. All the European journals have extolled, in the moſt enthuſiaſtic ſtrain, the perſon who devoted his labour to a matter of ſo much utility; but, notwithſtanding his valuable reſearches, his name is now ſeldom repeated. Yet how many unfortunate victims of a treacherous element might be ſtill in exiſtence, had they provided themſelves with preſervatives ſo eaſy to be obtained?

The Abbè de la Chapelle, being on the point of undertaking a long voyage, employed himſelf in endeavouring to find out a method by which the ſeamen might be ſaved, if by one of thoſe unfortunate events, too common at ſea or in rivers, they ſhould be obliged to abandon their veſſel, and to commit themſelves to the waves, in order to attempt reaching the ſhore by ſwimming.

With this view he invented a kind of ſwimming dreſs, to which he gave the name of *ſcaphander*. It is a ſort of jacket compoſed of pieces of cork, covered on each ſide with cloth, and faſtened round the body by means of leather thongs, which paſs between the thighs and over the ſhoulders.

That the body of the ſwimmer may be in equilibrium with an equal volume of water, a jacket of this kind ought to contain about ten pounds of cork.

The inventor of this apparatus tried it in the Seine during the bathing ſeaſon. Having put it on, he committed himſelf to the moſt rapid part of the river, where he kept his body in an upright poſture, with his head above water, and ſo much at his eaſe, that he could make uſe of a bottle and a glaſs which he held in his hands.

The ſcaphander, which may be procured at a ſmall expence, is ſuſceptible of farther improvement, according to the taſte and ingenuity of thoſe who may chooſe to ſuperintend the conſtruction of it. Beſides the utility of it in caſes of ſhipwreck, it will enable people to bathe, without danger, in places of a lake or river more ſalutary than at the edges, where one is frequently ſurrounded with filthy and diſguſting water.

This happy invention brings to our remembrance another, ſomewhat ſimilar, for the purpoſe of tranſporting infantry over rivers, where there are no bridges, and which cannot be forded. It conſiſted of a ſort of dreſs which ſupported the ſoldier exceedingly well in the water. The inventor, who was an officer, made a trial of it himſelf, and having been rowed out in a boat to a confiderable diftance at fea, threw himfelf into the waves, and returned to the fhore, walking in the water with as much eafe as if he had been on dry land.

Hydroftatic Balance.

This balance is an inftrument invented for the purpofe of determining the fpecific gravity of bodies. The conftruction of it is founded on a theorem of Archimedes, which fhews, that a body placed in water, lofes as much of its weight, as is equal to that of the volume of the liquid difplaced by it; from which it follows, that if we take the weight of the body in water from its weight in air, the difference will be the weight of a mafs of water equal to that part of the folid which is immerfed. This balance therefore is of great utility for determining the degrees of every kind of alloy; the quality and richnefs of metals, ores, and minerals, and the proportions of every mixture whatever.

Abfolute gravity is fo peculiar to a body, that it is always the fame, being proportioned to the quantity of matter it contains; but the fpecific gravity is the ratio of the weight of two bodies, having the fame volume. Thus, if a piece of cork weighs the hundredth part of a piece of lead of the fame volume, we may fay that the fpecific gravity of cork is to that of lead as 1 to 100.

To determine the fpecific gravity of a liquid, take a ſolid body of any form at pleaſure, and bring it into equilibrium in the air at the arm of the hydroſtatic balance, in order to aſcertain its abſolute gravity; then immerſe it entirely in the liquor, by which means the equilibrium will be immediately deſtroyed; and what you are obliged to add to reſtore it will be exactly the weight of the volume of the liquor diſplaced by the immerſed body. If the body be a cubic inch, and if after immerſion it has been neceſſary to add 4 drams, you may conclude that a cubic inch of the liquor weighs 4 drams. In experiments of this kind, the greateſt care muſt be taken that the ſolid and the liquor in which it is immerſed do not vary in their denſity during the operation; for in that caſe the reſult would not be accurate. It is on the ſame principles that areometers, for determining the different ſpecific gravities of liquors, are conſtructed.

Of the ancient philoſophers, Archimedes ſeems to have made the greateſt progreſs in the ſtudy of hydroſtatics. Having obſerved one day while bathing, that as he immerſed his body more or leſs, it diſplaced a greater or leſs volume of water, he was ſo ſtruck with this phenomenon, though apparently of little importance, that he hurried from the bath, and ran through the ſtreets of Syracuſe, crying out "I have found it; I have found it." When he returned to his ſtudy, he began to reaſon on this obſervation; and at laſt deduced from it certain principles, which enabled him to discover, by means of the hydrostatic balance, the quantity of alloy mixed in the crown of Hiero.

This great man, according to Vitruvius, had been requested by that prince to find out the quantity of silver contained in a crown of gold which he had ordered to be made; but in relating this fact, Vitruvius does not tell us the quantity of the gold, nor inform us of the reasoning which Archimedes employed to discover the deception of the goldsmith. We may however suppose, that the crown weighed 20 pounds; that when immersed in water it displaced 13; that a mass of pure gold of the same weight as the crown displaced only 12, and that a mass of silver displaced 18. From these data it will be found, by employing the rule of allegation, the principles of which we have already explained, that the goldsmith had mixed with the gold of the crown, $3\frac{1}{3}$ pounds of silver.

The hydrostatic balance furnishes us with the certain means of determining whether a piece of money be adulterated, and whether a diamond be real or false.

Paper, which when written on, the characters shall be Invisible.

Mix up some hog's lard very intimately with a little Venice turpentine, and rub a small portion of it, gently and in an equal manner, over very thin paper, by means of a piece of a fine ſponge.

When you are deſirous to employ this preparation for writing ſecretly to a friend, lay the above paper over that which you intend to diſpatch, and trace out whatever you think proper with a blunted ſtyle, by which means the fat ſubſtance will adhere to the ſecond paper in all thoſe places where the ſtyle has paſſed. The perſon who receives the letter may eaſily render it legible by ſprinkling over it a little coloured duſt, or ſome pounded charcoal well ſifted.

Method of employing the above kind of paper for tracing out with great eaſe all ſorts of figures.

Mix with the above compoſition ſome very fine lamp-black, and rub it gently over a piece of very thin paper; then wipe it carefully, ſo that when laid upon a ſheet of white paper, and preſſed down gently with the hand, no ſtain may remain upon the latter.

Having laid over this black paper, the print which you wiſh to copy, and placed white paper below it, by emploping a ſtyle you may transfer all the outlines of the print to the latter paper. The caſe will be the ſame if a piece of fine ſilk or linen be employed; and by theſe means it will be eaſy, even for thoſe ignorant of drawing, to paint flowers on any kind of ſtuff. After the outlines have been traced out, nothing will be neceſſary but to ſhade them with the proper colours, made ſufficiently liquid, and laid on in ſo light a manner, that they may not run or become ſcaly, when the cloth is expoſed to moiſture.

Secret Writing by means of Ciphers.

There are ſeveral methods of carrying on a ſecret correſpondence by writing, ſo that no perſon into whoſe hands the letters may fall ſhall be able to read them. The moſt uſual, and at the ſame time the eaſieſt, conſiſts in employing, inſtead of common ink, different kinds of liquids, which leave no ſenſible traces on the paper, but which have the property of becoming viſible when expoſed to heat, when immerſed in water, or when beſprinkled with ſome powder. Reſpecting theſe proceſſes, we have already ſpoken. The other method is that called commonly writing in ciphers. It may be varied almoſt without end; and though it cannot be demonſtrated that it is impoſſible to decypher it, the operation for that purpoſe may be rendered ſo tedious and laborious, as to make it exceedingly difficult.

This method in general conſiſts in ſubſtituting, for the letters of the alphabet, different ſigns agreed upon by the perſons who correſpond. When the ſigns which denote the ſame letters are always invariably the ſame, it is certain that they may be eaſily decyphered, eſpecially if the writing be in a known language; but when they are changed in ſuch a manner, that the ſame may denote different letters, or that the ſame letter may be indicated by different ſigns, the combinations which muſt be made in order to aſcertain their relation become ſo difficult, that it is almoſt impoſſible to diſcover their meaning.

Thoſe who write in cyphers, have always before them an alphabet of this kind agreed upon by the parties, which enables them to tranſcribe their letters, and to write the anſwers. This alphabet is called the key; and it is often ſo difficult to be diſcovered, that it requires long and tedious combinations. The following examples will be ſufficient to give an idea of this method of correſponding in ciphers.

The Myſterious Dial-Plate, or the Prudent Secretary.

Trace out on a ſquare piece of paſte-board a ſort of dial-plate, accurately divided into twenty-ſix equal parts, in which you muſt inſcribe the twenty-four letters of the alphabet, and the two conſonants J and V. Over this dial-plate place a circular piece of paſte-board, made to move on the centre of the former, and of ſuch a ſize, that the lines by which the former is divided, being prolonged, may mark out upon it an equal number of diviſions: in theſe diviſions inſcribe alſo the letters of the alphabet; but it is to be obſerved that it is not neceſsary to arrange them in their proper order as in the former.

It may readily be conceived that when the moveable circle has been placed in such a manner, that one of its divisions or letters corresponds with one of the other, all the rest of the divisions will correspond also.

When you intend, by means of this instrument, to write a letter in ciphers to a correspondent, who must be provided with one exactly similar, arrange the moveable circle in such a manner, that the divisions of both parts shall stand exactly opposite to each other; and that your correspondent may arrange his instrument in the same way, write at the head of the first line the letter *a*, with that corresponding to it in the opposite division; for example *am*, which will enable him to decypher your letter.

Thus, if the moveable circle is disposed in such a manner, that *w* of the first corresponds to *o*, and *e* to *r*; instead of *we*, you must write *or*, and you must continue in this manner, in regard to the letters which compose all the words you have occasion to write.

The person to whom you write will employ the index *a m*, as already explained, to enable him to dispose the moveable part of his instrument in the same manner; and by finding out successively all the letters which correspond to those of the inner circle, as they occur in his letter, he will decypher it ſpeedily, and with great eaſe.

Writing in Ciphers, which appears to be a piece of Muſic.

This ſingular method of writing in ciphers, is the ſame in principle as that above deſcribed: but with this difference, that inſtead of letters, the interior circle ought to contain in its diviſions muſical notes, different from each other both in figure and poſition.

The perſon who writes, muſt take care to put at the head of his letter ſome character of the alphabet, with the correſponding note, in order that this ſign may ſerve as a key to point out the manner in which the two parts of the inſtrument ought to correſpond. By theſe means muſical notes may be ſubſtituted for all the letters of the alphabet, ſo as to expreſs every thing neceſſary to be tranſmitted to your correſpondent.

Method of Writing in Ciphers with Lattice Work.

This method is both eaſy and expeditious: nothing is neceſſary but to provide a piece of paper, cut out with ſquare holes in regular order, according to the direction of the lines, and correſponding to thoſe of another piece of paper in the poſſeſſion of your correſpondent. If this paper be laid over another of the ſame ſize, write on the latter, through the holes, whatever you choose, and then, removing the upper paper, fill up the intervals with words forming some kind of sense with those before written on the paper.

Method of taking off Impressions in Plaster of Paris, or Sulphur.

As curious people, who cannot purchase the originals, are often desirous of obtaining impressions of medals, engraved stones, and other valuable articles preserved in cabinets, they may easily be procured, and at a very small expence. The whole process consists in a very simple operation, which will give a striking resemblance of the object, so as to exhibit all its parts with the greatest truth.

When you intend to take off an impression in plaster, that which has been pulverised and sifted through a piece of very fine silk must be employed. First rub over the medal, or engraved stone, very softly with oil, and having wiped it with cotton, surround the edge of it with wax or with a bit of thin lead: mix up the sifted plaster with water, and stir it gently, to prevent it throwing up air bubbles; then pour it over the medals, and suffer it to harden and dry. It may then easily be detached, and will form a mould, strongly marked, by means of which you may take off impressions in relief, either in plaster or sulphur. *

* Before these moulds are used, they must be impregnated with oil T

The procefs for melted fulphur, is the fame as for plafter: but it is to be obferved, that when the model is of marble, old lard ought to be employed in preference to oil, becaufe the latter, by penetrating through the pores of the marble, would ftain it.

Baits for Catching Fifh.

In order to attract fifh when angling, baits made of various kinds of grain, fuch as wheat, barley, oats, or boiled beans, mixed with aromatic herbs, and pounded with earth, may be employed. Fifh are wonderfully attracted by ftrong fmelling fubftances, as camphor, afafœtida, &c.: they feem to have a great fondnefs for a pafte made of crufts of bread, honey and afafœtida. It is faid alfo that they approach coloured objects through curiofity. Some people tie a bit of fcarlet rag to the hook, and rub it over with petroleum; and others highly extol heron-oil. To obtain the latter, the flefh of the heron is cut fmall, and pounded in a mortar; it is then put into a long necked bottle, clofely corked, and preferved for two or three weeks in a warm temperature; the flefh, by putrefying, is converted into a fubftance that approaches near to oil, which is mixed up with honey, bread and a little mufk. Moft fifh, and particularly carp, are faid to be very fond of this bait. Artificial infects are much ufed alfo for catching fifh, efpecially trout: they are made of different colours, according to the hours of the day, in order that they may imitate the natural objects which appear at these different periods.

Those who fish in fresh water, employ cheese sometimes as a bait, and prefer that which emits the strongest smell. The putrid livers and flesh of animals of every kind are likewise used.

Small, long, slender worms, of a white or pale yellow colour, with a red head, contained in small cells found in the roots of the water iris, are said to be excellent bait for trout, tench, carp, and various other kinds.

Earth worms, as well as those engendered in meat, are of great service.

To procure the latter, and almost at every season, a dead cat, or bird of prey, must be exposed to the flies, and when the worms become very lively it ought to be buried in moist earth, as much sheltered from the frost as possible. The worms may be taken out according as they are wanted. As these worms are metamorphosed into flies towards the month of March, recourse must then be had to other animals of the like kind.

Method of producing Variety in the Colours of Flowers.

Variety is generally produced in flowers by sowing, in the same bed, seeds collected from different individuals; and there is reason to think that this variety in colour, arises from the farina of the differently coloured flowers; which mutually fecundate each other.

This conjecture is supported by experience; for it is found, that if flowers of the same kind, but different in colour, that is some red and others yellow, flower together, the seeds arising from them produce red, yellow, and orange flowers, and even some diversified with red and yellow. It is certain also that the variegations of flowers, are more singular, according as the variety of colours contrasted together in the same bed, is greater; that by planting together, in the same pot, yellow and white ranunculuses, the seed resulting from them will produce sulphur-coloured ranunculuses; and that aurora-coloured ones may be obtained, in like manner, by a similar process, with yellow and red ranunculuses.

It may easily be proved by experiment, that this phenomenon arises only from the influence of the farina; because, when these flowers are planted separately, and at a distance from each other, they produce only the same colours.

Method of obtaining Double Flowers.

The more petals a flower has, it becomes the fuller and more beautiful. Flowers sometimes are converted into double ones by accident; but there are ſome which are only very little ſo, as may be obſerved among carnations. There is however an artificial method of making them become double, which is, to transplant them ſeveral times the firſt year, as in ſpring and autumn, without ſuffering them to flower. By following this method for two years conſecutively, ſingle carnations may ſometimes be converted into double ones.

Method of obtaining Flowers of Different Colours, on the Same Stem.

Scoop out the pith from a ſmall twig of elder, and having ſplit it lengthwiſe, fill each of the parts with ſeeds that produce flowers of different colours. Surround them with earth, and then tying together the two bits of wood, plant the whole in a pot filled with earth properly prepared. The ſtems of the different flowers will thus be ſo incorporated, as to exhibit to the eye only one ſtem, throwing out branches covered with flowers analogous to the ſeed which produced them. By ſelecting the ſeeds of plants, which germinate at the ſame period and which are nearly ſimilar in regard to the texture of their ſtems, an intelligent floriſt may obtain artificial plants exceedingly curious.

Crows.

These birds, though exceedingly uſeful on account of the havoc they make among certain inſects that deſtroy the corn, multiply ſo faſt, in ſome countries, that they do much miſchief, particularly among the game.

In the country, eſpecially during a fall of ſnow, the catching of crows may afford excellent amuſement. One method of catching them is as follows. Scrape *nux vomica*, and having rolled ſmall morſels of fleſh in it, ſcatter them about on the ground: the crows, attracted by the fleſh, will devour it, but they will ſoon after become intoxicated, and fall down as if dead. They will however recover from their intoxication, and if not ſpeedily caught may fly away.

Nux vomica is a mortal poiſon to dogs, which cannot be cured but by making them ſwallow vinegar.

As crows are exceedingly voracious, and fond of large beans, if a certain quantity of them ſtuck through with ſmall pins and needles be ſcattered about on the ground, from which the ſnow has been removed, their inteſtines will be ſo lacerated by them that they will ſoon die.

Crows may be caught alſo by putting bits of fleſh or beans into paper cornets, and then rubbing over the edges of the cornets with bird-lime: the cornets adhering to the heads of theſe animals will ſo blind them, that when they attempt to fly off, they will fall into the hands of thoſe who are waiting to catch them.

AIR BALLOONS.

This ingenious invention made a rapid progreſs in a very ſhort time, and indeed the caſe could not be otherwiſe at a period when natural philoſophy was cultivated by people of every rank, who mutually exerted themſelves to contribute towards bringing the diſcovery to perfection.

We ſhall not here ſpeak of the experiments which have been made on a large ſcale; nor of the dreadful cataſtrophe that befel two ærial argonauts, who acquired celebrity at the expence of their lives: the ærial voyages which have been undertaken might appear fabulous to poſterity, were they not well authenticated in different works, which appeared about that time.

It muſt however be allowed, that this diſcovery was purſued with great enthuſiaſm as long as it was believed that it could be of utility; but the ſize neceſſary for theſe balloons, the expence they occaſioned, the danger of their catching fire and ſuddenly burſting, the impoſſibility of directing them, and the ſmall weight they were able to ſupport, ought to have ſhewn that they could ſerve for little more than experiments, which, though ſurpriſing, were dangerous to thoſe who attempted them.

There are two kinds of balloons, and two methods of filling them: one is by heating, and thereby dilating, the air contained in them; the other is by filling them with inflammable air, which is much lighter than atmofpheric air.

The firft kind are made of linen, and the fecond of filk, done over with gum, when intended for experiments on a large fcale; but if they are deftined merely for amufements, it will be fufficient to make the former of paper, and the latter of gold beaters leaf: that we may not deviate from the object of this work we fhall here fpeak only of fmall balloons.

Paper Balloons.

As it is neceffary that thefe balloons fhould be exceedingly light, filk paper, which is very thin, and at the fame time not brittle, and which weighs no more than a dram per fourth of a fquare foot, is employed: the leaft fize that can be given to them, is four feet in diameter, otherwife they would be fpecifically heavier than the atmofphere, and would not rife. In order to give them a fpherical form, when feveral fheets of this paper have been cemented to the ends of each other, they muft be cut into the fhape of a fpindle, like thofe pieces ufed for covering terreftrial globes.

When cut into this fhape, join them two and two, and cut off, at one extremity the length of about 12 or 15 inches; then cement them all together into one ſpherical body, and border the aperture with a ribbon, leaving the ends, that you may ſuſpend from them the following lamp.

Conſtruct a ſmall baſket of very fine wire, if the balloon is ſmall, and ſuſpend it in the aperture, ſo that the flame of a few leaves of paper, wrapped together and dipped in oil, may heat the inſide of it. Before you kindle the paper, ſuſpend the balloon in ſuch a manner, that it may in a great meaſure be exhauſted of air, and as ſoon as it has been dilated, let it go, together with the wire baſket, which will ſerve it as ballaſt. Balloons of this kind will ſometimes go the diſtance of two or three leagues, ſupported in the air at a very high elevation, as long as the paper continues burning. This amuſement ought never to be practiſed when there is reaſon to apprehend that there are any combuſtibles in the neighbourhood, which the balloon may ſet on fire; though in general ſuch balloons do not fall till the flame is extinguiſhed. *

Balloons of Gold-beater's Leaf.

Theſe balloons may be made much ſmaller than thoſe of paper, not only becauſe they are much

* These experiments ought never to be performed when corn is on the ground, even though there may be none in the neighbourhood. T.

lighter, bulk for bulk; but because the inflammable air, with which they are filled, has less weight than air dilated by heat; yet they ought never to be less than a foot in diameter.

The gold-beater's leaf ought to be doubled, and when cut into the proper form, as above, should be cemented together with fish glue, in such a manner, as to leave no other aperture than a small tube for introducing the inflammable air. You may easily discover whether the pieces have been closely cemented, by introducing into it atmospheric air, which will not fail to escape if it finds any vent. Nothing is then necessary but to fill the balloon with inflammable instead of atmospheric air, which is specifically heavier than the former, in the proportion of 8 to 1. For this purpose, you must employ a large glass flask, which, besides the common neck, has one also in the side. Adapt to the upper neck a bent tube, the lower end of which must be placed in a bason of water to cool the gas in its passage through it. Over the orifice of the tube place an inverted funnel, and insert the tube of it into the orifice of the balloon. When every thing has been thus arranged, introduce into the flask, through the lateral neck, four or five drams of the filings of very pure iron, and a sufficient quantity of spirit of vitriol, mixed with one third of water. The inflammable air, extricated by the effervescence, will pass through the tube and the water into the funnel, by which means it will be conveyed into the balloon, and, being much lighter than atmofpheric air, will gradually expel the latter from it, and affume its place: the balloon will thus become fufficiently light to rife fpontaneoufly into the atmofphere.

The apparatus above defcribed, is fufficient for filling balloons 12 or 15 inches in diameter; but if they are large, like thofe of Blanchard or Pilatre-de-Rozier, &c, the apparatus muft be of a proportionable fize.

As the form of balloons is a matter of indifference, provided the air they contain, added to the weight of the fubftance which confines it, be lighter than an equal volume of atmofpheric air, they may be made to reprefent flying dragons, winged horfes, &c. But in thefe cafes, the amufement they afford is of fhort duration, as they foon get out of fight: to render this amufement agreeable, they ought to maintain themfelves at fuch a moderate elevation, that they can be eafily diftinguifhed.

To collect the Inflammable Air of the Marfhes.

Take a common bottle full of water well corked, and having made a round hole in the cork, introduce into it the tube of a glafs funnel, feven or eight inches in diameter, in fuch a manner, that the water in the bottle can iffue from it only through that tube. Invert the bottle in the ftagnant water, which covers some marsh, and stir the mud round the funnel by means of a sharp pointed stick, or rod of iron: inflammable air, having the same qualities as that above mentioned, will then rise into the bottle.

THE MAGNET.

The magnet is a metallic stone, generally of a greyish or a blackish colour, compact, and exceedingly ponderous, which is found for the most part in iron mines. It affects no peculiar form, and externally has nothing that distinguishes it from the meanest productions of the bowels of the earth. But its property of attracting or repelling iron, and of pointing to the north, when at freedom to move, gives it a distinguishing rank among the most singular objects of nature.

This stone, strictly speaking, is only a sort of iron ore, but belonging to that class called poor, because it contains but a small quantity of metal. Metallurgists have indeed been able to extract iron from it; but, besides its being difficult of fusion, the metal is in such small quantity, that it would not pay for the labour of working it.

Why then is not every kind of iron ore magnetic? This is a queſtion which, in my opinion, has never yet been anſwered. The reaſon of this, no doubt, is ſome peculiar combination of the iron with certain heterogeneous bodies. It is not improbable that it may contain ſome principle, which does not enter into the compoſition of the other ores of that metal; but it is not impoſſible that chemiſtry may one day diſcover in what the combination conſiſts.

The firſt diſcovery of the ſecrets of nature does not always unveil all their wonders. In general, we are able to penetrate into cauſes and effects only by deep reſearch, and repeated experiments. Such has been the caſe with the magnet: only one quality, that of attracting iron, was at firſt obſerved in it; and it was not till towards the 14th century that its other properties, and particularly that of turning towards the poles, and of communicating its virtue to ſteel, were diſcovered. This happy diſcovery, one of the moſt important of modern times, gave riſe to the invention of the mariners compaſs, without which it would have been impoſſible to undertake long ſea voyages.

But the magnet, like many other extraordinary things, has given riſe to ridiculous tales and fables filled with falſehoods and abſurdities. We are told, for example, that Mahomet's coffin, ſaid to be of iron, is ſuſpended in the air by the attractive virtue of a magnet. This fable, contradicted by all thoſe who have been at Mecca and Medina, is founded on what Pliny relates of the architect Dinocrates, who propofed conftructing at Alexandria a temple arched with magnets; in order that he might fufpend from them the tomb of Arfinoe, the fifter of Ptolomy, whom that prince wifhed by thefe means to immortalife; the death of Ptolemy, and that of Dinocrates, prevented the execution of this chimerical project.

Among thefe fables ought to be claffed what is related by Serapion, who pretends that the magnets found in great abundance in the bowels of the earth, ftop veffels in full fail, and attract the nails with which they are conftructed, fo as to make them drop out.

Various authors have afcribed to it, virtues ftill more marvellous, by pretending that it has the power to expel demons; and it is not long fince fome enthufiafts maintained that the magnetic fluid, the nature of which will perhaps never be known, is able to cure different maladies.

Famianus Strada however advances a ftill greater abfurdity, when he tells us, that by the virtue of this mineral, it is poffible to correfpond with our friends, even when at a great diftance. This ridiculous fiction, revived a few years ago, is indeed founded on fome probability; for we are taught by experience, that by means of a magnet two friends, in different chambers, may correfpond together, even when feparated by a partition half a foot thick; but this is impoffible at a greater diftance, or beyond the fphere of the magnetic attraction.

It is certain that the magnet has long been employed in various amufements, more or lefs agreeable, and which to fome have appeared to border on the marvellous. As a proof of this, we fhall mention a fact related by St. Auguftine, who tell us, that being one day at the houfe of a bifhop, named Severus, he faw him take a magnet, and hold it below a filver plate, containing a bit of iron, which exactly followed all the movements of the hand that held the magnet, and put it into motion: he then adds, that at the time he was writing, he had before him a veffel filled with water, placed on a table, and that a needle fufpended on the water moved from one fide to the other, according to the motion which he gave to a magnet under the table.

We are informed by Baptifta Porta in his *Magia naturalis*, that fome jugglers of his time took advantage of this property of the magnet to impofe on the credulity of the populace, by having a bafon infcribed round the edge with different words, which ferved as anfwers to queftions propofed to them by the fuperftitions, refpecting future events.

Souchi de Rennefort fays, in his work, among the tricks exhibited to the populace by jugglers and mountebanks, it is common for them to fhew a magnetic needle concealed in a piece of cork, floating in a bason of water, which without being touched by any one, moves up and down according to the movement of a magnet, held below the bason, and directed by the hand.

The same thing is done at present, but in a more ingenious manner. In the cabinet of that learned mechanic the Marquis de Servieres was a clock, having in its centre a bason filled with water; and an artificial tortoise, placed in the bason, always pointed out the hours, being moved by means of a magnet properly adapted. Something of the same kind is mentioned by the Abbe Nollet, who speaks of a clock, where the hours were indicated by an iron fly, which followed the motion of a loadstone, concealed under the dial-plate.

We have heard also of a syren, which by an application of the magnetic virtue different ways, produced amusements which exhibited a very extraordinary appearance.

These amusements having been attended with great success, on account of the wonder they excited, different kinds of them, more or less agreeable, all founded on the same principle, have been invented. But it would be erroneous to believe that they have given rise to the discovery of any new property in the magnet. All these wonderful effects are produced by properties which have been long known;

and are the refult of a different application of the fame principles.

Direction of the Magnet.

A magnet, or magnetic needle, fufpended freely on a pivot, always directs one of its ends towards the north, and the other towards the fouth. The magnetic matter which, according to the opinion of various philofophers, flows without intermiffion from the one pole of the earth to the other, gives this direction to the magne', or magnetic needle. Huygens accounts for it, by confidering the fmall particles of the magnetic matter as fo many fmall darts, and the pores of the magnet compofed of a multitude of fmall tubes, the interior furface of which is furnifhed with inclined flexible parts, always ready to rife up and oppofe the return of the magnetic matter.

Magnetic Attraction.

If the north pole of a magnet, or magnetic bar of iron, be prefented to the fouth pole of a magnetic needle, the needle will be attracted, and join itfelf to the magnet. The magnetic fluid, which iffues with great velocity from one pole of each of thefe bodies, finds free ingrefs into that of the other.

On the other hand, if the fouth pole of a magnet be prefented to the fame pole of a magnetic needle, the latter recedes, turns round, and becomes agitated, until it presents to it the north pole. The same thing takes place when the north pole of the magnet is presented to the same pole of the needle: it recedes, in like manner, and at last presents to it the south pole. Thus the poles distinguished by different names, attract each other, and those of the same name repel each other: the former are called friendly poles; the other hostile poles. These effects can neither be prevented nor even diminished by the interposition of any body, except iron.

Communication of the Magnetic Virtue.

If a needle, or a thin plate of well tempered steel, be drawn gently over the north or south pole of an armed magnet, or magnetic bar of iron, moving it from the one end to the other, always in the same direction; this needle or plate will also become a magnet, having poles, and the same virtue as the magnet itself.

If the needle be afterwards moved in a contrary direction, over the same pole of the magnet; it will immediately lose its virtue; but if you continue to move it in the same direction, it will resume its virtue, with this difference, that the south pole becomes the north pole, and the north one the south.

Needles, or plates of steel, rendered magnetic by the above process, will retain their virtue for several years, though it gradually decreases; but this decrease is often occasioned by neglecting to keep them always in the direction of the magnetic fluid, which proceeds from one pole to the other, or by their acquiring ruft, which lessens their magnetic property: it may, however, be restored by polishing, and again subjecting them to the same operation.

Declination of the Magnetic Needle.

We have already shewn, that a magnetic needle, freely suspended, always directs itself towards the poles; but this direction varies several degrees, and this variation is called the needle's declination.

The declination is not the same in every part of the globe; and it even varies at different times in the same place; sometimes it is east, and sometimes west. The cause of this variation has never yet been discovered, and it would be useless to enlarge on it here, as it has no relation with philosophical amusements, the singularity of which depends only on the magnetic virtue.

Inclination of the Magnetic Needle.

The inclination of the needle, is the tendency it has, when suspended, to depress itself towards the poles. A steel rod suspended on an axis, in the same manner as a balance, and brought into perfect equilibrium before the magnetic virtue is communicated to it, does not preserve that equilibrium after it has been rendered magnetic. It seems then to be more ponderous on the ſide towards the neareſt pole, and it inclines towards it in proportion as the latitude increaſes; at London, for example, it inclines about 60 degrees towards the north; if the ſame needle were removed to the equator it would be in equilibrium, and placed beyond it it would incline towards the ſouth.

The ſame effect takes place when a needle is held ſuſpended in a ſimilar manner above, or is placed by the ſide of a magnetic bar of iron: if it be placed towards the middle, or what repreſents the equator of the bar, it will remain in a horizontal poſition parallel to the bar; and it inclines or approaches it more and more, according as it is removed from that place, and advanced towards either of the poles of the magnetic bar; ſo that if the bar be much longer than the needle, when the extremity of the needle comes near that of the bar, it will place itſelf in a vertical direction, that is to ſay, perpendicular to the bar. It muſt here be obſerved, that in this experiment, the axis muſt paſs through the needle in a horizontal direction, when it is placed above the bar, in order that by theſe means it may have full liberty to incline itſelf.

On the Choice of Magnets.

The beſt, and moſt valuable magnets, are thoſe which, being of equal ſize, are capable, after they have been armed, of raiſing the greateſt weight:

the difference between them is ſo great, that the attractive force of ſome is a hundred times as great as that of others. It is very common to find magnets which, when armed, can raiſe five or ſix times their weight: but their ſtrength is rarely increaſed ſo much as to be able to raiſe a hundred times their weight.

Whatever ſtrength theſe armed magnets may have, they are ſcarcely of any uſe for communicating the magnetic virtue. Bars of iron or artificial magnets, communicate the magnetic virtue much better, and by their help a very great number of bars of ſteel may be rendered magnetic, which could not eaſily be done by the beſt magnet. For the method of communicating the magnetic virtue to bars of every ſize, we are indebted to Mr. Knight, who made known his proceſs to the Royal Society.

Method of Conſtructing and Magnetiſing Bars, and Bundles of Bars, neceſſary for communicating the magnetic virtue to Artificial Magnets, which may be employed for various Amuſements.

Cauſe to be forged a dozen of ſteel bars, eight inches in length, ſeven or eight lines in breadth, and two in thickneſs; poliſh them on the ſides, and let the two extremities be filed exactly ſquare. After they have been thus prepared, bring them to a red heat throughout in the fire, and then temper them, without making them too hard; obſerving, at the ſame time, every neceſſary precaution to prevent them from being bent.

When theſe bars are well tempered, they muſt be again poliſhed by means of a grindſtone, and by applying them afterwards to a ſtone of a ſofter nature.

Before they are tempered, care muſt be taken to mark, by a notch with a file, that ſide of the bar deſtined to become the north, in order that there may be no miſtake when they are magnetiſed.

When this operation is finiſhed, join the twelve bars together, by means of two rings of copper, taking care to ſeparate them by a ſmall wooden rule, and to place ſix on the one ſide of it, and ſix on the other, in ſuch a manner, that their extremities may be in alternate order, that is to ſay, a north and a ſouth always placed together.

When they are thus arranged, and well faſtened together by means of the copper rings, poliſh them again by a cutler's wheel and emery. Then mark the order in which they are arranged, that you may be able to place them in the ſame manner when they are magnetiſed; and it is of eſſential importance that the ends of none of them project beyond the other.

Cauſe to be made alſo two plates of ſoft iron, of the ſame breadth as the bundle of bars, which may cover all their extremities, and let them be half an inch in thickneſs: Theſe plates will adhere ſtrongly to the magnetic bars, and contribute to preserve their virtue longer. To one of these plates may be affixed a hook, for the purpose of suspending from it some weight, which will tend to increase the magnetic force, provided the burthen it has to support does not gradually increase as the force is augmented.

Remove the rings which keep the bars together, and arrange them end to end, six of them at a time in the same line, and in such a manner that similar poles may not correspond; then draw a good artificial magnet along the whole line, observing that the side of the magnet which ought first to pass over it must be of the same name as the pole corresponding with the extremity of the line; that is to say, if the south pole, for example, first presents itself, the south pole of the magnet must be the first drawn over the bars.

When the artificial magnet has been drawn over them ten or twelve times, backwards and forwards alternately, the same operation must be performed on the other side of them.

Then take one of these bars, and try to suspend from one of its extremities another of them, by presenting their contrary poles to each other; if one of them raises another, and the latter a third, they are sufficiently magnetised; and by forming them into a bundle, they may be employed for communicating magnetic virtue to the other six.

As the latter six will have more ſtrength than the former, they ought to be uſed for magnetiſing the former a ſecond time; if, notwithſtanding this precaution, you find that they continue weaker, it will be of no avail to endeavour to magnetiſe them farther, as the deficiency muſt be owing either to the quality of the ſteel, or to ſome fault in the temper.

When you are well aſſured that the bars have been ſufficiently magnetiſed, they muſt be bound cloſely together in a bundle as before, and armed with their plates above mentioned.

Such magnets are ſufficient for communicating the magnetic virtue to bars of eight or ten inches in length; but for bars of eighteen or twenty inches, the bundle employed ought to conſiſt of a greater number.

It is to be obſerved, 1ſt. That when a bundle is formed, there muſt always be an even number of bars, divided into two equal portions by a ſlip of wood, two lines in thickneſs. 2nd. That the bars on the one ſide muſt lie in a contrary direction to thoſe on the other; that is to ſay, if the one have always their ſouth poles turned upwards, the others muſt have theirs turned downwards.

Method of Communicating the Magnetic Virtue to a Bar of Steel, without the help of a natural or an artificial magnet.

Provide a plate of untempered ſteel, about three inches in length, three or four lines in breadth, and half a line in thickneſs. Then take a ſhovel and a pair of tongs* (the more they have been uſed, and the larger they are, ſo much the better); hold the ſhovel vertically between your two knees, and fix in it the plate of ſteel in ſuch a manner, that the extremity deſtined for the north pole may be turned downwards: in order that it may not be diſplaced, it will be proper to tie it to the ſhovel with a ſilk ſtring. Then take the tongs, and, holding them nearly in a vertical direction, rub the plate with their extremities, proceeding always from the bottom upwards. When this operation has been performed twelve or fifteen times, on both ſides of the plate, it will have acquired ſufficient magnetic virtue to raiſe ſmall nails by its lower extremity. This diſcovery was made by Mr. Canton.

It may be readily conceived that when ſix or eight plates have been magnetiſed, they may be formed into a ſmall bundle, for communicating the magnetic virtue to larger ones, which may be employed to give the like property to others of ſtill greater ſize.

How to Diſcover the Poles of a Magnet.

If a magnet he held horizontally over a piece of

* Fire irons which usually stand at the side of the chimney answer better than those which are allowed to lie on the fender. T.

paper, covered with iron filings, you will obſerve two places almoſt diametrically oppoſite to each other, where the filings are more crowded, and where the ſmall oblong fragments ſtand on their ends, as it were, while in every other part they lie on their ſides.

This experiment will enable you to diſcover the poles of the magnet. Every magnet indeed has two poles, or two oppoſite points, which, as we ſhall ſhew hereafter, poſſeſs different and peculiar properties. One of theſe points is called the north pole, and the other the ſouth; becauſe, if the magnet be freely ſuſpended, the former will turn of itſelf towards the north, and the other conſequently will point towards the ſouth. Theſe points muſt be aſcertained in a magnet, before any experiments are made with it.

The production of New Poles in a Magnet, and one Pole Changed into Another.

If a magnet, or a magnetic bar, be cut through in a direction perpendicular to its axis; that part of it towards the north pole of the magnet, or bar, will acquire a ſouth pole, and the other will acquire a north pole.

The poles may eaſily be changed by the following experiments. If you preſent to a needle, freely ſuſpended in water, a magnetic bar, the needle will be attracted by it; but if the bar be ſubjected cold to a violent blow, by ſtriking it in a perpendicular direction on an anvil, the needle will be repelled by the ſame end that before attracted it; which could not be the caſe unleſs the one pole were converted into the other.

The recreations founded on the magnetic virtue, are too extenſive to be introduced into this collection. What we have ſaid will be ſufficient to exerciſe the curious, and to enable them to invent amuſements themſelves, or, at any rate, to diſcover the pretended myſtery of thoſe exhibited by jugglers.

THE LEARNED SPANIEL.

A learned ſpaniel, which maintained philoſophical theſis in Engliſh, French, and Latin, was exhibited about twenty years ago at York. It may readily be conceived that the animal did not ſpeak theſe languages; but he ſeemed, at leaſt, to underſtand them, ſince, if aſked any queſtion in them, he always replied by ſigns; either ſhaking his head to expreſs yes or no; or pawing with his foot to indicate numbers or letters, which when joined together formed the required anſwer.

Three circumſtances occurred to excite the aſtoniſhment of the ſpectators, who were attracted in great numbers by the celebrity of this animal. 1ſt. He continued to give pertinent and proper anſwers, even when his maſter retired from the exhibition-room, or deſired all thoſe to retire who were ſuſpected of making ſigns to the dog to indicate the anſwer. 2nd, He returned anſwers equally proper when blindfolded, to prevent him from obſerving any ſigns. 3d, He generally advanced the moſt ſingular paradoxes: At firſt no perſon in company agreed with him in opinion, yet aſter a variety of objections, anſwers and replies, he was always allowed in the end to be right.

To prove that the epithet given to this learned animal, was not altogether misapplied, we ſhall here relate a kind of converſation which took place between the ſpaniel and two or three learned perſons in company.

A ſailor firſt aſked him how many arches there were in Weſtminſter-bridge. The ſpaniel replied, by drawing his foot over the number 15. He was then aſked how many arches there were in the Pontus Euxinus. Here the dog pauſed, as if he had conceived himſelf inſulted by ſuch a queſtion, and as if deſirous of applying the proverb, " a fooliſh queſtion deſerves no anſwer." Being commanded however by his maſter, to ſatisfy the perſon who had interrogated him, he replied, that the Pontus Euxinus had no arches, and he expreſſed this very clearly by placing his foot on a cypher. The ſailor then ſaid that the preceding year he had made a very happy voyage, in ſix weeks, from the Pontus Euxinus to London-bridge. The ſpaniel, finding nothing very wonderful in ſuch a voyage, placed his foot on different letters, forming a very laconic anſwer, which ſignified, when explained by his maſter, that ſome navigators had made a voyage of 600 leagues in half a day. "That is impoſſible," ſaid the ſailor, "no air balloon has ever yet been able to traverſe ſuch a ſpace in ſo ſhort a time." "I do not ſay," returned the ſpaniel, by the help of his interpreter, "that an air balloon was employed for that purpoſe: I ſpeak of a voyage by ſea."

The ſailor then ſaid, that by ſea it was ſtill more impoſſible, becauſe as the faſteſt ſailing veſſel went at the rate of no more than about five leagues per hour, it could never make a voyage of 600 leagues in half a day.

The animal perſiſted in maintaining its aſſertion, and the ſailor was going to lay a conſiderable bet, when the ſpaniel and his maſter added, that they had performed this voyage in a country where they had kindled fire with ice.

"If you are deſirous of ſhewing your erudition," replied the ſailor, "do not, I beg of you, utter ſo many abſurdities." The maſter of the ſpaniel then addreſſing the animal, ſaid: "Tell us, my friend, is it not true that a fire can be kindled with a piece of ice, if it be cut into the form of a lens, ſo as to collect the ſun's rays into a focus, and to project them on a ſmall heap of gun-powder?" The animal, which was blindfolded, nodded with his head, to ſay yes; as if he had fully comprehended the queſtion propoſed to him.

"The dog, on this point, is right" ſaid the ſailor; "but it does not prove that a journey of 600 leagues can be performed in half a day." "Why not," replied the dog by the mouth of his maſter, "if it be in a country where in half a day there are 360 hours." "In what climate," ſaid the ſailor, much ſurpriſed, and beginning to perceive the truth of his reply. The ſpaniel mentioned the frigid zone. "In that zone," ſaid his maſter, "the days indeed are of different lengths, from 24 hours to ſix months. If Captain Cook," added he, "when he ſailed beyond the polar circle had followed a parallel, where the day was only a month long, he might, in half a day conſiſting of 360 hours, have traverſed the ſpace of 600 leagues."

The ſailor being deſirous to difficult the ſpaniel and his maſter, in his turn, aſked them if they knew a place where the ſun and moon might riſe at the ſame hour, and even at the ſame inſtant, when these two luminaries are in opposition, that is to say at full moon. The animal and his master replied, that it was at the pole; adding, that in the same place the sun was always on the meridian, because every point of the horizon was south to the inhabitants, if any, at the pole.

A lawyer, who was present, disputed a long time against the spaniel, because the latter pretended that a man who died at noon, might sometimes be the heir of another, who died the same day at half an hour after twelve. Though various laws were quoted from the Digesta and the Justinian code, which declare that the heir must survive the testator, the spaniel proved that the assertion was perfectly agreeable to these laws; because the person who died at noon might, in certain circumstances, have survived him who died at half after twelve: this would be the case if the first died at London, and the other at Vienna.

A third person proposed the following problem. "A countrywoman having gone to market to sell chickens, met with a cook, who bought the half of what she had and the half of one more, without killing any of them. She then sold to a second cook the half of those remaining and half a chicken more, also without killing any, and afterwards the half of the remainder and half a chicken more to a third cook, still without killing any. By these means the country-woman ſold all her chickens: how many had ſhe?"

The ſpaniel replied that ſhe had ſeven: that the firſt purchaſer took four, that is to ſay three and a half plus one half, without killing any; that the ſecond had taken two, that is to ſay one and a half plus a half; and, in the laſt place, that the third had taken one, that is to ſay, one half plus a half.

It now remains for us to explain, how the animal, without any viſible ſign being made to him, could return anſwers to the queſtions propoſed to him. The reader muſt know, that the letters and figures were placed on ſo many pieces of card, arranged in a circular manner around the animal; that he moved round the circle as ſoon as any queſtion was propoſed, and that levers concealed under the carpet on which he walked, and which were made to move under his feet by means of ropes, indicated to him the exact moment when he ought to ſtop, to place his foot on the neareſt card. He was ſo well habituated to hit the card next to him when he felt the levers move, and to give an affirmative or negative anſwer by the motion of his head, according as his maſter or any confederate altered the tone of his voice, that he never once erred.

THE COOLING OF LIQUORS.

In warm countries, ſuch as India, Perſia, and other parts of Aſia, it is neceſſary, on account of the great heat, to deviſe means for cooling thoſe liquors uſed as beverage. Of all the methods hitherto employed, the moſt natural is, to ſurround the veſſels containing the liquor with ice; but as it is often impoſſible to procure it, the veſſel with the liquor to be cooled may be immerſed in water, in which ſome ſal ammoniac has been diſſolved: as this ſalt, of all thoſe ſoluble in water, is that which cools it moſt, it is exceedingly proper for cooling any kind of liquor uſed for drinking.

As the dearneſs of this ſalt, however, may often prevent it from being employed, recourſe may be had to the Indian method, which conſiſts merely in wrapping wet cloths round the bottles containing the liquor; expoſing them to a current of air, and moiſtening the cloths as they become dry. This proceſs will render the liquor ſufficiently cool to moderate the ſtifling heat which we ſometimes experience in our own climates.

This curious phenomenon is produced by the evaporation of the water with which the cloths are moiſtened; the more evaporable therefore liquors are, the more cold they produce. Water inclosed in the bulb of a small thermometer, may be congealed by wrapping round it a rag dipped in ether, the evaporation of which will be accelerated if it be agitated in a circular manner.

According to Chardin, there are some cities in Persia and Egypt, where the chief branch of trade consists in the sale of vessels made of a kind of porous earth, which promote the evaporation of a small quantity of water contained in them, and maintain coolness by means of the constant moisture which prevails on their exterior surface. Travellers, by means of these vessels, are always enabled to have the pleasure of drinking cool water.

By observing what substances, when mixed with ice, produce the greatest degree of cold, it has been found possible to freeze mercury. This experiment proves what before was only suspected, that mercury is a metal solid by its nature, but fusible at so low a temperature, that it always has a sufficient degree of heat to remain in a state of fusion.

Method of Cooling Liquors at Sea.

Captain Ellis, by the help of a small cask, constructed in such a manner, as to take up a quantity of sea water from any depth at pleasure, found that at a certain depth, the water was salter and considerably heavier. Water drawn up from the depth of a thousand fathoms, kept Fahrenheit's thermometer at 53 degrees; whereas that at the surface kept it at 84. People at sea, who are exposed to many inconveniencies, may in consequence of this discovery procure cool beverage, when in hot climates, by letting down to a certain depth in the sea, the vessels containing the liquors which they use.

ASTRONOMICAL PARADOXES.

The heavenly bodies are never seen in the place where they really are: for example, the whole face of the sun is seen above the horizon after he is set.

Though this has the appearance of a paradox it is a truth acknowledged by all astronomers, and which philosophers explain in the following manner:

The earth is surrounded by a fluid, to which we give the name of atmosphere. As the light by which we see the heavenly bodies, does not reach our eye but by continually passing from a rarer into a denser medium, it must necessarily come to us in a curved line. But as the apparent place of the objects will be to us in the prolongation of a tangent to this curve, we muſt therefore ſee them nearer the zenith than they really are; and this difference, between the real and the apparent place, will be greater the nearer the body is to the horizon, becauſe the rays then paſs through a greater ſpace of the atmoſphere.

Aſtronomers have found, that when the body is on the horizon, this refraction is about 33 minutes; therefore when the upper limb of the ſun is in the horizontal line, ſo that if there were no atmoſphere he would ſeem only beginning to peep over the horizon, he appears to be elevated 33 minutes, and as the apparent diameter of the ſun is leſs than 33 minutes, his lower limb will appear to touch the horizon. Thus the ſun is riſen in appearance, though he is not really ſo, and even when he is entirely below the horizon. Hence follow ſeveral curious conſequences, which it is proper to be remarked.

1ſt. More than one half of the celeſtial ſphere is always ſeen, though, in every treatiſe on the globes, it is ſuppoſed that we ſee only the half; for, beſides the upper hemiſphere, we ſee alſo a band round the horizon of about 33 minutes in breadth, which belongs to the lower hemiſphere.

2d. The days are every where longer, and the nights ſhorter, than they ought to be, according to the latitude of the place; for the apparent riſing of the ſun precedes the real riſing, and the apparent ſetting follows the real ſetting; therefore though the quantity of day and night ought to be equally balanced at the end of the year, the former exceeds the latter in a conſiderable degree.

3d. The effect of refraction, above deſcribed, ſerves alſo to account for another aſtronomical paradox, which is as follows:

The moon may be ſeen eclipſed even totally and centrally, when the ſun is above the horizon.

A total and central eclipſe of the moon cannot take place, but when the ſun and moon are directly oppoſite to each other. We here ſuppoſe that the reader is acquainted with the cauſes of theſe phenomena, an explanation of which may be found in every elementary work on aſtronomy. When the centre therefore of the moon, totally eclipſed, is in the rational horizon, the centre of the ſun ought to be in the oppoſite point; but by the effect of refraction theſe points are raiſed 33 minutes above the horizon. The apparent ſemi-diameter of the ſun and moon, being only about 15 minutes, the lower limbs of both will appear elevated about 18 minutes.

Such is the explanation of a phenomenon which muſt take place at every central eclipſe of the moon; for there is always ſome place on the earth where the moon is on the horizon at the middle of the eclipſe.

4th. Refraction enables us to explain also a very common phenomenon, viz, the apparent elliptic form of the ſun and moon when on the horizon; for the lower limb of the ſun correſponding, we ſhall ſuppoſe with the rational horizon, is elevated 33 minutes by the effect of refraction; but the upper limb being really elevated 30 minutes (this being nearly the apparent diameter of that luminary at its mean diſtances) is elevated in appearance by refraction no more than 28 minutes above its real altitude; the vertical diameter, therefore, will appear ſhortened by the difference between 33 and 28, that is to ſay 5 minutes; for if the refraction of the upper limb were equal to that of the lower, the vertical diameter would be neither lengthened nor ſhortened. The apparent vertical diameter will thus be reduced to 28 minutes. But there ought to be no ſenſible decreaſe in the horizontal diameter, for the extremities of this diameter are carried only a little higher in the two vertical circles paſſing through them, and which, as they meet only in the zenith, are ſenſibly parallel. The vertical diameter then being contracted, while the horizontal diameter remains the ſame, the reſult muſt be, that the diſks of the ſun and moon will apparently have an elliptical form.

5th. There is always more than one half of the earth enlightened by a central illumination, that is to ſay, by an *illumination* the centre of which is viſible; for if there were no refraction, the centre of the ſun would not be ſeen till it correſponded with the plane of the rational horizon; but as the refraction raiſes it about 33 minutes, it will begin to appear when it is in the plane of a circle parallel to the rational horizon and 33 minutes below it.

There is therefore a central illumination for the whole hemiſphere, plus the zone comprehended between that hemiſphere and a parallel diſtant from it 33 minutes; and there is a *complete illumination* from the whole diſk of the ſun to the ſame hemiſphere, and the zone comprehended between the border of it and a parallel about 18 minutes below the horizon.

ANIMAL MAGNETISM.

There are a thouſand pretended ſecrets, which have no other foundation than the avarice of a few needy enthuſiaſts, the blind zeal of their diſciples, and the ſtupid ignorance of thoſe who are fond of novelty. When ſubjected to the impartial teſt of reaſon and experiment, they vaniſh like ſubſtances incapable of ſtanding the proof of chemical analyſis.

The univerſal panacea, the unextinguiſhable lamp, the perpetual motion, the malleability of glaſs, the philoſophers ſtone, and the quadrature of the circle, no longer preſent themſelves to the mind but as objects which expoſe to ridicule thoſe who have ſacrificed their time or property in the purſuit of theſe chimerical diſcoveries. Even air-balloons, ſo much boaſted of throughout Europe, ſeem to have been buried in the ſame grave along with Pilatre-des-Roziers.

Animal Magnetiſm, which Meſmer and his dupes found means to bring into ſome credit, exiſts no longer but in the imagination of a few quacks, ſince a learned commiſſion applied the torch of reaſon and accurate inveſtigation to its pretended wonders.

Extract from the Report of the Commiſſioners appointed by Louis XVI. to examine Animal Magnetiſm.

On the 12th of March 1784, this prince appointed Meſſrs. Borie, Lallin, Darcet, and Guillotin, members of the faculty of Paris, to examine the animal magnetiſm, practiſed by M. Deſlon, and to give in a report to him on the ſubject; and in conſequence of a requeſt made by theſe phyſicians, his Majeſty appointed five members of the Academy of Sciences, Meſſrs. Franklin, Leroi, Bailly, de Bory, and Lavoiſier, to aſſiſt them in this examination. As M. Bory died when the commiſſioners began their labours, his Majesty made choice of M. Majault, member of the faculty of medicine, to succeed him.

Explanation of the Doctrine of Animal Magnetism.

The agent which M. Mesmer pretends to have discovered, and which he has made known under the name of animal magnetism, is a fluid diffused throughout the whole universe; it is the means of a mutual influence between the celestial bodies, the earth and animated beings; it is continued in such a manner as to leave no vacuum; its subtlety is beyond all comparison; it is capable of receiving, propagating and communicating all the impressions of motion, and is susceptible of a flux and reflux. Animal bodies experience the effects of this agent, and it is by insinuating itself into the substance of the nerves that it immediately affects them. The human body, in particular, possesses properties analogous to those of the magnet, and it has also its different and opposite poles.

The action and virtue of animal magnetism may be communicated from one body to other bodies, either animate or inanimate, and even at a considerable distance, without the aid of any intermediate body: it is increased and reflected by glass; it is communicated and propagated by sound; in a word, this virtue may be accumulated, concentrated, and transmitted. Though the fluid be universal, all animated bodies are not equally susceptible of it; there are some even, though few in number, of a nature so hostile to it, that their presence alone destroys all the effects of it in other bodies.

Animal magnetism can cure immediately all diseases of the nerves, and others it cures mediately; it strengthens the action of medicines, and it excites and directs salutary cures in such a manner, that they may be overcome. By its means, the physician can ascertain each individual's state of health, and can speak with certainty respecting the origin, nature, and progress of the most complicated diseases; he can prevent their increase, and be able to cure them without ever exposing the patient to dangerous effects, or disagreeable consequences, whatever be the age, the temperature, or the sex. In animal magnetism, nature presents an universal medium for curing and preserving mankind.

Such is the agent which the commissioners were charged to examine, and such the properties warranted by M. Deslon. This physician, when he explained to the commissioners the doctrine and nature of animal magnetism, taught them also the practise, and made them acquainted with the poles, by shewing them the method of touching the patients, and of conveying to them the magnetic fluid.

Description of the Treatment.

After having procured information reſpecting the theory and practice of animal magnetiſm, it was neceſſary to ſee its effects: for this purpoſe, the commiſſioners attended, and each of them ſeveral times, to obſerve the manner in which M. Deſlon treated his patients. In the middle of a large hall they ſaw a circular box made of oak, raiſed to the height of a foot, or a foot and a half, which was called the tub; the upper part of this box was pierced with a great number of holes, from which proceeded iron branches, having moveable elbows. The patients were placed in rows around this tub, each oppoſite to one of the iron branches, which by means of the elbow could be applied directly to the diſeaſed part. They were all united to each other by a rope that went round their bodies; and ſometimes they formed a ſecond chain, by laying hold of each others hands: that is to ſay, by each applying the thumb between the thumb and fore-finger of the next perſon.

In a corner of the hall ſtood a piano-forte, on which various airs were played in different time; and theſe airs were ſometimes accompanied with the voice and ſinging.

All the magnetiſed perſons held in one of their hands an iron rod, about ten or twelve inches in length.

M. Deslon declared to the commissioners, 1st. That this rod was the conductor of magnetism; that it possessed the property of concentrating it at its point, and of rendering its emanations more powerful. 2d, That sound, according to the principles of Mesmer, was also a conductor of magnetism: and to communicate the fluid to the piano-forte, nothing was necessary but to bring the iron rod near it; those who touched the instrument furnished it also, and the magnetism was transmitted by the sounds to the surrounding patients. 3d, The rope which went round the patients, as well as the joining of hands, was destined to increase the effects by communication. 4th, The inside of the tub was constructed in such a manner, as to concentrate the magnetism; it was a large reservoir, from which it was propagated by means of the iron branches fixed in it.

Method of Executing and Directing the Magnetism.

The patients arranged in great numbers, and in several rows, around the tub, received the magnetism at the same time by all these means: by the iron branches which transmitted to them the magnetism in the tub; by the rope twisted round their bodies; by the joining of hands, which communicated to them that of their neighbours; and by the sound of the piano-forte, or of an agreeable voice. The patients were magnetised also directly by means of the finger, and of an iron rod moved before the face, or behind the head, and on the diseased parts, always observing the distinction of the poles: the person who performed the operation acted upon them also by the look, and by staring at them; but they were magnetised in a particular manner by applying the fingers to the hypochondriac region, and sometimes continuing to do so for several hours.

Effects observed on the Patients.

The patients exhibited a highly varied picture, according to their different states. Some of them were calm and tranquil, and experienced no effects whatever; others coughed, spat, and felt some slight pain, a local or a universal heat, and fits of perspiration; others were agitated and tormented with convulsions, which were remarkable for their number, their duration, and violence. As soon as one convulsion began, several others manifested themselves. The commissioners saw some which lasted three hours; they were accompanied with the expectoration of a turbid viscous liquor, forced up by the violence of the efforts. Sometimes streaks of blood came up; and one young man in particular, one of the patients, threw up a great deal of it. These convulsions were characterised by sudden and involuntary movements of all the limbs, or contraction of the throat, subsultus of the hypochondria and the epigaſtrion uneaſineſs, a wildneſs of look, piercing cries, weeping, hiccup, and immoderate laughter; they were preceded and followed by a ſtate of languor and reverie, a ſort of dejection, and even lethargy. The leaſt unexpected noiſe produced in the patients a tremor, and it was obſerved that changing the tone and meaſure of the airs, played on the piano-forte, had an influence on them; ſo that by livelier tunes they were more agitated, and the vivacity of their convulſions was renewed.

A hall, lined with matting, had been at firſt deſtined for patients afflicted with theſe convulſions, and on that account was called the *chamber of criſes*; but M. Deſlon does not think proper to make uſe of it, and all the patients, whatever be their ſymptoms, are collected together in the public hall.

Nothing can be more aſtoniſhing than the appearance exhibited by theſe convulſions: none but thoſe who have ſeen them can form any idea of them, and thoſe who ſee them are not a little ſurpriſed at the profound tranquility of one part of the patients, and the agitation experienced by another; the various ſymptoms which are repeated, and the ſympathetic emotions produced. Some patients attach themſelves to each other excluſively; ruſh towards each other, laugh, addreſs each other in an affectionate tone, and mutually ſoften each others criſes. They are all ſubject to him who magnetiſes;

though in an apparent ſtate of ſtupor the ſound of his voice, a *look*, or a ſign, is ſufficient to rouſe them from it, and in conſequence of theſe conſtant effects, it is impoſſible not to acknowledge ſome great power by which the patients are agitated and ſubdued; and of which he who magnetiſes ſeems to be the depoſitory.

Concluſion.

The commiſſioners having aſcertained, that this animalo-magnetic fluid cannot be perceived by any of our ſenſes; that it has no action either on them, or on the patients ſubjected to it; and having aſſured themſelves that preſſure and touching occaſion changes ſeldom favourable in the animal economy; and, in the laſt place, having demonſtrated by deciſive experiments, that the imagination without magnetiſm produces convulſions, and that magnetiſm without the imagination produces nothing, have unanimouſly concluded, in regard to the queſtion of the exiſtence and utility of the magnetic fluid, that nothing proves its exiſtence; that the violent effects obſerved on thoſe ſubjected to public treatment, ariſe from touching; from the imagination being put in action, and from that mechanical imitation which impels us in ſpite of ourſelves to repeat whatever ſtrikes our ſenſes. At the ſame time they think themſelves obliged to add, as an obſervation of importance, that frequent touching, and the repeated action of the imagination to produce crises, may be prejudicial; that the sight of these crises is equally dangerous, on account of that imitation which nature seems to have imposed on us as a law; and consequently that all public treatment, where the means of magnetism are employed, must in the end produce fatal effects.

At Paris, March 11th, 1784.

(Signed) BENJAMIN FRANKLIN, MAJAULT,
LEROI, SALLIN, DE BORY,
BAILLY, D'ARCET, GUILLOTIN,
LAVOISIER.

We have thought it necessary to enlarge so much on the doctrine, management, and effects of the pretended *magnetic fluid*, and have given at full length the opinion of the learned commission who examined it, in order to undeceive those who, under the pretence of a recreation might become the dupes of a practice dangerous to society.

WEIGHTS FOR BALANCES.

Examination of the number and relation of weights, with which any number of pounds, from one to any determinate number, may be weighed in the simplest manner.

Though this, on the firſt view, may appear to belong to mechanics, it may be readily ſeen that it is nothing but an arithmetical problem; for the queſtion may be reduced to this. To find a ſeries of numbers, beginning with unity, and which added or ſubtracted from each other in every manner poſſible, ſhall form all the numbers from unity, up to any number required.

This queſtion may be reſolved two ways, viz. by addition alone, or by addition combined with ſubtraction. In the firſt caſe, the ſeries of weights, which anſwers the problem, is that of the numbers increaſing in a double progreſſion; in the ſecond, that of the triple progreſſion.

Thus, for example, if we have weights equivalent to 1, 2, 4, 8, and 16 pounds, we may weigh, by means of theſe, any number of pounds up to 31; for with 2 and 1 we can form 3 pounds; with 4 and 1, 5 pounds; with 4 and 2, 6 pounds; with 4, 2 and 1, 7 pounds; &c. With one more weight of 32, we can weigh 63 pounds; and ſo on, doubling the laſt weight and deducting unity from that double.

But if weights in the triple progreſſion be employed, as 1, 3, 9, 27, 81, by means of theſe we can weigh any number of pounds from 1 to 121; for with the ſecond leſs the firſt, that is to ſay, putting the firſt into one ſcale and the ſecond into the other, we make 2 pounds; by putting both into the same scale we form 4 pounds; by putting 9 into the one side, and 3 + 1 into the other, we form 5 pounds; 9 on the one side and 3 on the other, will give 6 pounds; 9 + 1 on the one side and 3 on the other, will make 7 pounds; and so by different transpositions, we may proceed in this manner to the weight above mentioned.

It is here evident that the last method is simpler than the other, as it requires a less number of different weights.

These two progressions are, in this respect, more advantageous than any arithmetical progression that could be tried; for if weights in the natural arithmetical progression 1, 2, 3, 4, &c, were employed, to weigh 120 pounds would require 15: and to weigh 121 pounds, with weights in the following arithmetical progression 1, 3, 5, 7, &c, would require 11. As no other but the triple progression would make up all the numbers possible, from the weight of one pound to the greatest resulting from the whole of the weights, the triple progression is therefore the most advantageous.

It is evident that the solution of this problem is of considerable utility in the ordinary affairs of life, since it affords the means of weighing any number of pounds, with the least number possible of different weights.

PERPETUAL LAMPS.

Before the improved ſtate of philoſophy had ſhewn the impoſſibility of real un-extinguiſhable fire, the learned were much divided in their opinions on this ſubject; but of all the champions in favour of perpetual lamps, none has made greater efforts to obtain credit to their exiſtence, than Fortunio Liceti, in his book entitled *De Reconditis Antiquorum Lucernis*.

If we can believe this author, nothing was more common among the ancients than perpetual lamps. The lamp of Demoſthenes, that which burnt in the temple of Minerva at Athens, the Veſtal fire at Rome, all furniſh him with ſo many proofs of the poſſibility of un-extinguiſhable fire. One cannot help ſmiling to ſee ſo much learning ſo badly employed: for who is there who does not know that theſe fires were called perpetual merely becauſe it was a point of religion to preſerve them from being extinguiſhed, and to ſupply them with continual aliment.

The other partiſans of perpetual lamps, while they laugh at the ſimplicity of Liceti, ſupport their reaſoning on facts which ſeem to carry with them a little more weight; they are as follow:

1. *The Lamp of Tulliola.*

The tomb of Tulliola, the beloved daughter of Cicero, and whose death cost him so many tears, was discovered, it is said, under the pontificate of Paul III. It is pretended, that in this tomb there was a lamp actually burning, but which became extinguished on the admission of air.

2. *The Lamp of Olybius.*

But it is the lamp in the tomb of Olybius, which above all others, supplies the partisans of perpetual lamps with one of their strongest arguments.

In the year 1500, as we are told, some peasants digging the earth to a considerable depth at Atesta, in the neighbourhood of Padua, came to a tomb, in which they found two earthen urns, one within the other. The latter, it is added, containing a burning lamp, placed between two phials, one filled with liquid gold, and the other with liquid silver. Both these urns were ornamented with Latin inscriptions.

Such is the manner in which several authors relate this curious discovery. But what follows is still stronger: Liceti gives a letter of one Maturantius, who tells his friend Alphenus, that he had got possession of this incomparable treasure. "Both the vases," says he, "with the inscriptions, the lamp and the phials, have fallen into my hands, and are now in my possession. If you should see them you would be astonished: I would not part with them for a thousand crowns of gold." This is no doubt the language of a man who believes he possesses a valuable rarity. We do not however know that it exists in any collection.

It appears, that in this case, as at the tomb of Tulliola, an accident prevented enlightened people from being witnesses to the phenomenon; for we read in the credulous Porta, that as the peasants who found this treasure handled it too roughly, the lamp broke in their hands, and was extinguished.

3. *The Lamp of Pallas, the son of Evander.*

We are told also, that about the year 800 of the Christian æra, the tomb of the famous Pallas, the son of Evander, killed as is well known by Turnus, was found at Rome.

Within it was a burning lamp, which consequently must have burnt nearly 2000 years, since it was shut up in the year 1170 before the Christian æra.

4. *The Lamp in the Temple of Venus.*

Various celebrated authors pretend, that this lamp burnt perpetually, and that the flame adhered so ſtrongly to the combuſtible matter, that neither wind, rain nor tempeſts could extinguiſh it, though continually expoſed to the air, and to the inclemency of the ſeaſons. The partiſans of this lamp, after inſinuating that a wick of aſbeſtos was, in all probability, employed, conclude by ſaying, that it might perhaps have been the work of the devil, in order to blind the pagans more and more, and to attach them to the infamous deity worſhipped in that temple.

5. *Lamps of Caſſiodorus.*

Caſſiodorus, who it is well known, was as much celebrated by his employments as by his talents, tells us himſelf that he made perpetual lamps for his monaſtery at Viviers. Each monk, it is probable, had one of them for his own uſe. His words are: "We have provided for our vigils lamps which always retain their light, and the oil of which is never conſumed, though conſtantly expoſed to the ardour of the flame."

Some partiſan of perpetual lamps may here ſay: "Is it poſſible to refuſe credit to teſtimony ſo authentic, ſo clear, and ſo reſpectable?"

Such are the principal facts adduced in favour of perpetual lamps; but we may venture to ſay, that they will not ſtand the teſt of critical examination. In regard to the firſt three, what dependance can be placed on facts related in so vague a manner, and accompanied with incoherent and romantic circumstances? None of these facts are supported by any other testimony, than that of men who lived a long time after; no person, whose authority is of any weight, asserts that he actually saw them. But in disputes respecting things which are contrary to the common laws of nature, they must at least be certified by enlightened men, above all suspicion of credulity or ignorance.

The tale respecting the tomb of Tulliola, is as old as the year 1345; a period when all Europe was sunk in the grossest ignorance. A body is said to have been found in it, and in that case it could not be the body of Tulliola; for the Romans in the time of Cicero always burnt their dead. In consequence of this and similar circumstances, some authors have conjectured, that the tomb alluded to, was that of the wife of *Stilico*; but the Christians never placed lamps in their tombs. The account therefore of a lamp found in this tomb has every appearance of a fiction.

But what shall we say of the tomb of Olybius, and the lamp with two phials, one filled with fluid gold, and the other with fluid silver? This double urn was found by peasants, who according to some authors, handled the lamp contained in the second urn so clumsily, as to break it; and yet Maturantius pretends that he had it in his possession. Who ſaw the lamp burning? What evidence have we that the peaſants ſaw it in that ſtate? and whoſe teſtimony in this caſe would be admiſſible? Some vapour exhaled from a place ſhut up for ſo many centuries might eaſily impoſe on rude and ignorant people.

The ſtory of the tomb of Pallas, the ſon of Evander, is ſcarcely worthy of refutation. To ſhew the impoſture, nothing is neceſſary but to read the Latin inſcription, ſaid to have been contained on it, which is as follows:

> Filius Evandri Pallas quem lancea Turni
> Militis occidit, more suo jacet hic.

Who will be ſo weak as to believe, that theſe verſes were written in the time of Eneas? One needs only to have ſeen the language of the twelve tables, to be able to judge how little reſemblance the ancient language of the Romans, and that conſequently of the period of the Kings of Alba, bore to theſe Latin verſes. In regard to the lamp of Venus, we ſhall obſerve that thoſe who have ſpoken of it, do not ſay that it was not ſupplied with new aliment. What ſeems to occaſion the greateſt difficulty, is, that it could not be extinguiſhed, either by wind or by rain; but in this there is nothing wonderful, ſince our oilmen ſell flambeaus which have the ſame property. Beſides, even admitting that this lamp was perpetual, and inextinguiſhable, who is ſo ignorant as not to know, that the Pagan prieſts were the greateſt of *impoſtors*, and that they might employ many artifices to ſupply the lamp with new aliment?

The lamps of Caſſiodorus may be explained with equal eaſe; they were lamps which, like thoſe of Cardan, ſupplied themſelves with oil by means of a reſervoir; and Caſſiodorus only meant to ſay, that theſe lamps laſted a long time, in compariſon of the common lamps of that period, which ſtood frequently in need of having oil poured into them.

Theſe reflections did not eſcape ſeveral ingenious philoſophers, and particularly Octavio Ferrari, to whom we are indebted for a curious and learned work, *De Veterum Lucernis ſepulcralibus*. All theſe authors overturn the arguments of Liceti, and fully ſhew that the facts adduced in favour of perpetual lamps, reſt on a weak foundation; and that they abound with abſurdities and contradictions. They even ridicule the weakneſs of this learned man, who by an exceſs of credulity, almoſt beyond belief, finds in the lamp of the tomb of the necromancer Merlin, deſcribed by the poet Arioſto, a proof of perpetual lamps.

We ſhall conclude this article with the following very juſt reflections of Octavio Ferrari, above mentioned, which naturally ſuggeſt themſelves to the mind. If the ſecret of preparing perpetual and inextinguiſhable fire, had been known to the ancients, would an art ſo uſeful have remained buried in oblivion? But even admitting that it might be loſt for want of philoſophical and chemical knowledge, is it poſſible that Pliny, who enumerates the common inventions, as well as thoſe moſt celebrated, ſhould ſay nothing of this perpetual fire, a thing ſo wonderful? When Plutarch makes mention of the lamp of Jupiter Ammon, becauſe it burnt a whole year, is it to be ſuppoſed that he would obſerve ſilence reſpecting lamps, in compariſon of which the former was a contemptible trifle.

We muſt therefore ſay, that both hiſtory and ſound criticiſm oppoſe every idea of ſuch an invention having ever exiſted. We ſhall now examine how far it is conſiſtent with the principles of philoſophy.

On the Phyſical Poſſibility of making a Perpetual Lamp.

Having proved the weakneſs of all the facts brought as proofs in favour of perpetual lamps, it remains that we ſhould examine, how far they are poſſible, according to the principles of ſound philoſophy.

To obtain a perpetual lamp, it would be neceſſary to have as follows:

1ſt. A wick which could not be conſumed.

2d. Some aliment which could not be conſumed, or a ſubſtance which, after having ſerved as aliment to the fire, ſhould return into the veſſel without loſing its inflammable quality.

3d. It would be neceſſary alſo, that the flame ſhould be able to exiſt a long time in a place abſolutely cloſe, and of ſmall dimenſions; for ſuch were the tombs in which theſe perpetual lamps are ſaid to have been found.

But all theſe things are impoſſible, as will be ſeen by what follows.

§. I. *Impoſſibility of having a Perpetual wick.*

The curious properties aſcribed to the amianthus, which are in part real, are well known. We will not deny that a wick, capable of laſting for a very long time, might be made by means of this ſubſtance: but we muſt poſitively aſſert, that it would not be perpetual; for however much its incombuſtibility may have been boaſted of, this property is not abſolute; the fire would at length deſtroy it, as it does every other body. It is very true, that cloth made of the amianthus, when thrown into the fire, may be taken out ſound and entire, but not abſolutely ſo; for it is obſerved that each time it is expoſed to the fire, it loſes ſome of its weight. It would therefore be at length conſumed, and perhaps in a ſhort time, if it were only made red hot and then cooled, or if it were left a conſiderable time in a very ſtrong fire. A wick of amianthus would conſequently at the end of a certain time be completely deſtroyed.

Some have tried to make wicks of bundles of gold threads, exceedingly fine; but it has never been poſſible to light them, and even if this could be done, another inconvenience would prevent their being of any uſe: they would be fuſed by the flame, and conſequently would be rendered unfit for anſwering the intended purpoſe.

§. II. *Impoſſibility of procuring Indeſtructible Aliment for the perpetual lamps. Pretended recipes for making Indeſtructible Oil.*

But we ſhall even ſuppoſe a wick abſolutely unalterable to have been found, and that it does not become choaked up with fuliginous matter, from the combuſtible ſubſtance by which it is fed; this however would be only one ſmall part of what is neceſſary for obtaining a perpetual lamp: ſome kind of aliment which ſhall experience no diminution, or which having ſerved to maintain the flame without experiencing any alteration, may return by a perpetual circulation into the veſſel from which it iſſued, will alſo be requiſite. Is all this poſſible?

Let us now hear the alchemists, or the partiſans of perpetual lamps. We ſhall be entertained with their ideas reſpecting the manner in which an oil, ſuch as theſe lamps would require, might be obtained.

Some, considering that the amianthus resists fire, have tried, but without success, to extract an oil from it.

Others observing that gold and silver, but particularly the former of these metals, are indestructible by fire, conceived an idea of searching in them for that valuable oil, which would put us in possession of perpetual lamps. This is the noble secret with which Liceti pretends that Olybius was acquainted; but the metals are as incapable of producing oil as the amianthus.

It may however be said, that if it were possible to reduce gold to a liquid state, we might perhaps obtain an incombustible oil, as gold is unalterable in the fire. But besides the impossibility of converting gold into a liquid, how do we know that it would be indestructible?

The Abbot Trithemius, or the person who in his name has written a great many falsehoods, pretends to give a recipe for an unextinguishable oil, together with the whole process for making a perpetual lamp. Various alchemists have done the same; but the least knowledge of chemistry is sufficient to shew the ridiculous absurdity of these pretended discoveries.

§. III. *The impossibility of continually maintaining fire in a place absolutely close.*

It is a fact, well known to all those who cultivate natural philosophy, that if a lighted taper be placed under a glaſs receiver, ſo as to prevent its having any communication with the external air, the flame will gradually decreaſe in ſize, become faint, extend itſelf in length, and at laſt be extinguiſhed. Chemiſts have even calculated with great accuracy, what quantity of air is conſumed in a certain time by the combuſtion of a taper of certain dimenſions; ſo that we may eaſily predict in what time the flame will infallibly be extinguiſhed.

Beſides, it is well known that the cavities of the ancient tombs were exceedingly ſmall; and, to increaſe the difficulty, it is ſaid that the perpetual lamps burnt in the veſſels in which they were incloſed. Such at leaſt was the caſe with that of Olybius; but if this veſſel had been three feet in diameter, which by no means appears, it is certain that a lamp could not have exiſted in it two hours, without corrupting the whole of the air it contained, and without being extinguiſhed.

We ſhall not enlarge farther on this ſubject, as it would be waſting time to accumulate arguments to combat this chimera of perpetual lamps.

MERIDIAN LINE.

Of all the ſciences which form the object of human ſtudy, the moſt valuable are thoſe which tend to promote the convenience of mankind.

Aftronomy, which excites our aftonifhment, by making us acquainted with the fituation, order and motion of the different parts of the univerfe, befides furnifhing matter for the moft fublime fpeculation, is attended with this advantage, that it enables us to improve geography and navigation, to determine the annual revolution of the fun, and to avoid falling into error and confufion. Gnomonics, or the art of conftructing fun-dials, is deduced from this fcience; it makes us acquainted with the equality or inequality, and even the relation, of the different parts of the day; and by thefe means ferves as a rule, by which every thing can be performed at the proper time.

Machines, which human induftry has been able to carry to a ftate of perfection that could fcarcely have been expected, we here mean clocks, timekeepers, and watches, are indeed commonly employed for this purpofe; but thefe inftruments, however worthy of admiration, require to be regulated by a meridian. The philofopher by means of a certain line drawn in his chamber, can enjoy the fatisfaction of knowing the precife moment when the fun paffes the meridian, and can tell every day and every month how much the earth advances towards the fun; how much it afterwards recedes from it, and alfo thofe boundaries which it never paffes. A line of this kind may be drawn in a garden, on the wall of a houfe, or on that of an apartment.

Easy method of drawing a Meridian Line on a horizontal plane.

On a horizontal plane raise perpendicular to it a style or needle, or a metal peg, terminated by a plate having a hole in it, nearly about a line in breadth, and describe from the bottom of the vertical needle, or from the centre of the hole in the plate, several concentric circles. Then observe, both in the forenoon and afternoon, the points of the same circle which are touched at these different periods by the extremity of the shadow of the style, or by the centre of the light passing through the hole in the plate; draw a straight line through these two points, and divide it into two equal parts. When this is done, draw a straight line from the middle of the latter to the centre of the concentric circles, by which means you will have a meridian traced out with the utmost precision.

To draw a Meridian Line on the floor of an apartment.

In the window of an apartment, where you are desirous to trace out a meridian line, erect a style, terminated with a plate, having in it a hole, about two lines in diameter; that it may not be too much or too little raised above the floor, it will be proper, before it is fixed, to measure at noon the distance from the aperture of the window to the extremity of the chamber, following the direction indicated by the shadow which the edge of the window makes on the floor; this will give the length of the meridian, which we shall here suppose to be ten feet. Then fix the style in the bottom of the window, in such a manner, that the hole in the plate at its extremity may be elevated three feet two inches and a quarter above the floor. Watch the moment when a horizontal dial, of the accuracy of which you must be well assured, exactly indicates noon, in order that you may mark on the floor the place of the centre of the light which passes through the hole in the style; for this will be one point of the meridian. To find the second, necessary for marking its direction, you must extend a thread so as to form an inclined plane, from the hole in the plate to the point of the meridian already marked on the floor. Suspend from this thread a plummet, so far from the wall that no obstacle may turn it aside from its vertical direction, and mark on the floor the point exactly under the end of the plummet: if a line be then drawn from this point to that already found, it will be the meridian required.

Method of telling the hour of the night, on a sun-dial, by the light of the moon.

To employ a sun-dial as a lunar one, it must be observed, that this planet comes to the meridian at the same time as the sun only on the days of new and full moon. At these two periods, then, the moon ought to indicate the same hour as the sun; but except on these two days the light of the moon will make the sun-dial 48 minutes, or four-fifths of an hour, too late each day, on account of her own motion from west to east. Having therefore observed the hour indicated on the sun-dial, by the lunar rays, you must add to it 48 minutes taken as many times less one as the moon is days old, in order to obtain the hour as it would be indicated by the sun.

For example, if you find that the shadow of the style indicates six in the evening, on the sixth day of the moon, you must add to that hour four times four-fifths of an hour, which are equivalent to four hours; the result will shew that, according to the sun, it is ten o'clock at night.

To facilitate these numerical researches, we shall here add a table indicating the difference of the solar and lunar hours at the different ages of the moon. This table, which consists of two columns, exhibits on one side the days of the moon's age, and on the other the hours and minutes the moon is each day later than the sun. It is evident from what has been already said, that there ought to be no difference between the first day and the sixteenth, the second and the seventeenth, the third and the eighteenth, &c. In this table therefore the days, reckoning from the new moon, and those reckoning from the full moon, are placed oppofite to each other, fince the retardation of the moon, as compared with the fun, is not fenfible, but from thefe two periods.

To complete this table, the proper hours to be added muft be marked oppofite to the different days of the moon's age. Thus, for the fifth and the twentieth day of the moon, 3 hours 12 minutes muft be added to the hours indicated on that day by a fun-dial illuminated by the moon.

Days of the Moon.		Hours.	Minutes.
1	16	0	0
2	17	0	48
3	18	1	36
4	19	2	24
5	20	3	12
6	21	4	0
7	22	4	48
8	23	5	36
9	24	6	24
10	25	7	12
11	26	8	0
12	27	8	48
13	28	9	36
14	29	10	24
15	30	11	12

HYGROMETERS.

To measure the Moisture and Dryness of the air; short account of the principal Hygrometers invented for this purpose; their Faults.

Air is susceptible not only of acquiring a greater or less degree of heat, but also of becoming more or less moist. It is therefore one of the objects of natural philosophy to measure this degree of moisture, especially as that quality of the air has a great influence on the human body, on vegetation, and on a great many more of the effects of nature. This has given rise to the *hygrometer*, which is an instrument proper for measuring the humidity of the air.

But it must be allowed, that the instruments hitherto invented for this purpose, have not been attended with that accuracy which there was reason to expect. We have hygrometers indeed which indicate that the air has been more or less moist; but they have often this fault, that they shew a greater degree of moisture than really exists in the atmosphere: besides, they are not comparable; that is to say, it is not possible by their means to compare the moisture of one day, or of one place, with that of another. It may however be proper to describe these different hygrometers, were it only that their utility may be tried and examined.

1. As fir wood is very susceptible of participating in the dryness and moisture of the atmosphere, some have conceived the idea of applying this property to the construction of an hygrometer. For this purpose, a small, very thin fir board, is placed across between two vertical immoveable pillars, so that the fibres stand in a horizontal direction; for it is in the lateral direction, or that transversal to its fibres, that fir and other kinds of wood are extended by moisture. The upper edge of the board ought to have a small rack, fitted into a pinion, connected with a wheel, and the latter with another wheel, having on its axis an index. It may be easily perceived that by these means, the least motion communicated by the upper edge of the board to the rack, by its rising or falling, will be indicated in a very sensible manner by the index; consequently, if the motion of the index be regulated in such a manner, that from extreme dryness, to extreme moisture, it may make a complete revolution, the divisions of this circle will indicate how much the present state of the atmosphere is distant from either of these extremes.

This invention is ingenious, but it is not sufficient. The wood retains its moisture a long time after the air has lost that with which it was charged; besides, the board gradually becomes less sensible to the impressions of the air, and therefore produces little or no effect.

2. Suspend a small circular plate by a fine string, or piece of catgut, fastened to its centre of gravity, and let the other end of the string be attached to a hook. According as the air is more or less moist, you will see the small plate turn round, in one direction, or in another.

The hygrometers commonly sold, are constructed on this principle. They consist of a kind of box, the fore part of which represents a building with two doors. On one side of the metal plate which turns round, stands the figure of a man with an umbrella, to defend him from the rain, and on the other a woman with a fan. The appearance of the former of these figures indicates damp, and that of the other dry weather. This pretended hygrometer can serve for no other purpose than to amuse children; for the philosopher must observe that, as the variations of humidity are transmitted to this instrument only by degrees, it will indicate moisture or drought when the state of the atmosphere is quite contrary.

3. Some have tried to construct a hygrometer, by making fast a piece of catgut at one extremity, winding it backwards and forwards over different pullies, and suspending from its other extremity a small weight, behind which is placed a graduated scale. Others dispose the extremity of the catgut

in ſuch a manner, as to cauſe it to move an index round a graduated plate, the different degrees of which indicate the dryneſs or moiſture of the atmoſphere. This inſtrument however is ſubject to the ſame inconveniences as that before mentioned.

4. Put into one ſcale of a balance any ſalt that attracts the moiſture of the air, and into the other a weight, in exact equilibrium with it. The ſcale containing the ſalt will ſink down during damp weather, and thereby indicate that ſuch is the ſtate of the atmoſphere. An index, to determine the different degrees of drought or moiſture, may be eaſily adapted to it.

This inſtrument however is worſe than any of the reſt; for a ſalt immerſed in moiſt air becomes charged with a great deal of humidity; but loſes it very ſlowly when the air becomes dry: fixed alkali of tartar even imbibes moiſture till it falls in *deliquium*, that is to ſay, till it is reduced to a liquid or fluid ſtate.

5. Muſic may be employed to indicate the dryneſs or moiſture of the air. The ſound of a flute is higher during dry than during moiſt weather, and the ſtring of a violin exhibits the ſame phenomenon; but neither of theſe can ſhew the immediate ſtate of the air in regard to dryneſs or humidity.

From what has been here ſaid, there is reaſon to conclude, that no comparative hygrometer has ever yet been invented.

SNAILS.

That part of animals which has the external appearance of the head, ought not always to be confidered as the head, but that part only containing the brain, which is the general organ where all the fenfible parts that concur to promote animal life terminate. There are fome animals indeed which prefent in their anterior part certain organs, which might be taken for heads; but which however are fo only in appearance. Such are all infects in the ftate of larvæ: nature has placed in the anterior part of their body a round ring, in the form of a head, which they employ while in that ftate to receive and mafticate their food; and for that purpofe this organ is armed with a kind of forceps, in the fame manner as the real head of beetles. When the animal is transformed into a chryfolite, this ring detaches itfelf from it entirely; and it is then feen that it is not a real but a falfe head, united by nature to the phyfical conftruction of the infect in the ftate of larva. The cafe is the fame with fnails: in this aftonifhing animal the brain, from which the nerves proceed, is placed in the hind part of the neck, where it has the appearance of a grey ring, and the apparent head, which, in its natural pofition, is about five lines diftant from this ring, is nothing elfe than a prolongation of the neck or anterior extremity of the animal, in which nature has placed the organs of mastication, seeing and feeling.

According to these principles, which are the fruit of mature reflection on the internal structure of snails, the reproduction of the pretended head of these animals does not exhibit to the enlightened observer, either that singularity or importance which some have attached to it. The object of this phenomenon is only an extremity which, though it seems a head to the eyes of the vulgar, is far from appearing so to the eyes of the philosophic observer. To cut off therefore the anterior extremity of snails, is the same thing, in regard to the head, as to cut off the posterior extremity, or the end of the tail of a salamander, which will be seen to be reproduced.

But if this supposed head be cut off when the animal contracts itself, the brain being then less distant from the extremity, and as we may say in its place, it will be maimed by the operation; and in this case the animal, instead of reproducing the amputated part, will lose its life in a few instants. This is the reason why, of a hundred snails, the heads of which are cut off by an unskilful hand, very few of them reproduce that part; for by amputating the extremity which remains at the time of the animal's contracting itself, the operator cuts off a part of the brain, which really constitutes the head of the snail, and which cannot be injured without deſtroying the animal. On the other hand, if the apparent head be cut off when the animal is fully extended, the operation ſucceeds, and reproduction takes place. My curioſity has been ſeveral times excited by the ſingularity of this phenomenon, and I have repeated the experiment with ſo much ſucceſs, that I conſider it as eſtabliſhed on truth.

It follows from what has been here ſaid: 1ſt. That in organiſed bodies in general, both animal and vegetable, reproduction does not take place, but in parts merely acceſſory, and never in thoſe which have an immediate connection with their exiſtence, or which are eſſential to life; becauſe, in cutting off the latter, the ſources of reproduction are deſtroyed. 2d. That in mixt beings, the faculty of reproduction is always in the inverſe ratio of their perfection and ſenſibility; that is to ſay, the more complex the organic parts of an animal, and the ſtronger its powers of ſenſation, the leſs are its means of reproduction. Hence it happens that birds and quadrupedes, which are very perfect, and have great ſenſiblity, produce only parts void of ſenſation, ſuch as the feathers, claws, &c; and as there is little animal perfection in worms and ſnails, the want of ſenſation in which is ſupplied by muſcular irritability, they have the property of reproducing even irritable extremities, provided the brain, which is the ſource of all the ſenſible parts, remains untouched. In the laſt place, animals lefs complex in their organization, and which confift only of a repetition of fimilar parts, much more irritable than fenfible, reproduce themfelves entirely in every part of their body cut off, and are revived as we may fay in each portion of them, as is the cafe in regard to polypes.

By the help of thefe principles, deduced from the real theory of reproductions, general as well as particular, we may be readily convinced, that if an animal cannot reproduce thofe of its parts which are immediately connected with the principle of fenfation, much lefs will it reproduce a *real head*; that is to fay, the organ of the brain, from which is derived the principle that forms the effence of animal life.

TABLES OF CHANCES ON GAMES OR PLAY.

THE following tables contain the odds or chances, for winning any number of games, in a great variety of cases, either when the chance is equal in every game or throw, or when it is unequal according to any odds or proportion; and the games may be of any kind whatever, either at dice, or cards, or hazard, or billiards, or racing, or cocking, &c.

I. *When the chances or bets on each game are equal.*

Against winning		
21 times running	the odds are	2097151 to 1
20 out of 21,	are	95324 to 1
19 out of 21,	are	9038 to 1
18 out of 21,	are	1341 to 1
17 out of 21,	are	276 to 1
16 out of 21,	are	74 to 1
15 out of 21,	are	$24\frac{1}{2}$ to 1
14 out of 21,	are	$9\frac{1}{2}$ to 1
13 out of 21,	are	$4\frac{1}{5}$ to 1
12 out of 21,	are	2 to 1
20 games running,	are	1048575 to 1
19 out of 20,	are	49931 to 1
18 out of 20,	are	4968 to 1
17 out of 20,	are	775 to 1
16 out of 20,	are	168 to 1
15 out of 20,	are	$47\frac{1}{4}$ to 1
14 out of 20,	are	$16\frac{1}{3}$ to 1
13 out of 20,	are	$6\frac{1}{2}$ to 1
12 out of 20,	are $2\frac{9}{10}$, or very near 3	to 1
11 out of 20,	are 4 to 3, or	$1\frac{1}{3}$ to 1
even games in 20,	are 4s. 8d. to 1s. or	$4\frac{2}{3}$ to 1

19 games running, -	are -	524287 to 1
18 out of 19, -	are -	26213 to 1
17 out of 19, -	are -	2743 to 1
16 out of 19, -	are -	450 to 1
15 out of 19, -	are -	103 to 1
14 out of 19, -	are -	$30\frac{1}{3}$ to 1
13 out of 19, -	are	near 11, or $10\frac{9}{10}$ to 1
12 out of 19, -	are -	$4\frac{1}{2}$ to 1
11 out of 19, -	are 2s. 1d. to 1s. or near	2 to 1
18 games running, -	are -	262143 to 1
17 out of 18, -	are -	13796 to 1
16 out of 18, -	are -	1523 to 1
15 out of 18, -	are - -	264 to 1
14 out of 18, -	are - -	63 to 1
13 out of 18, -	are - -	$19\frac{3}{4}$ to 1
12 out of 18, -	are - -	$7\frac{1}{3}$ to 1
11 out of 18, -	are - -	$3\frac{1}{7}$ to 1
10 out of 18, -	are near 10 to 7, or nearer	$1\frac{5}{11}$ to 1
even games in 18, -	are	$4\frac{4}{5}$, or nearer $4\frac{7}{18}$ to 1
17 games running, -	are -	131071 to 1
16 out of 17, -	are - -	7280 to 1
15 out of 17, -	are - -	850 to 1
14 out of 17, -	are - -	156 to 1
13 out of 17, -	are - -	$39\frac{3}{4}$ to 1
12 out of 17, -	are	$12\frac{9}{10}$ or near 13 to 1
11 out of 17, -	are - -	5 to 1
10 out of 17, -	are -	13 to 6, or $2\frac{1}{6}$ to 1
16 games running, -	are	65535 to 1
15 out of 16, -	are -	3854 to 1
14 out of 16, -	are - -	477 to 1
13 out of 16, -	are - -	93 to 1

12 out of 16,	are	25 to 1
11 out of 16,	are	$8\frac{1}{2}$ to 1
10 out of 16,	are	$3\frac{1}{3}$ to 1
9 out of 16,	are	near 7 to 5, or nearer $1\frac{10}{21}$ to 1
even games in 16,	are	$4\frac{1}{11}$ or near 4 to 1
15 games running,	are	32767 to 1
14 out of 15,	are	2047 to 1
13 out of 15,	are	269 to 1
12 out of 15,	are	55 to 1
11 out of 15,	are	$15\frac{7}{8}$ to 1
10 out of 15,	are	$5\frac{1}{2}$ to 1
9 out of 15,	are	23 to 10, or $2\frac{3}{10}$ to 1
14 games running,	are	16383 to 1
13 out of 14,	are	1091 to 1
12 out of 14,	are	153 to 1
11 out of 14,	are	$33\frac{6}{7}$ or near 34 to 1
10 out of 14,	are	$10\frac{1}{8}$ to 1
9 out of 14,	are	$3\frac{2}{3}$ to 1
8 out of 14,	are	near 3 to 2, or $1\frac{1}{2}$ to 1
even games in 14,	are	near $3\frac{33}{43}$, or $3\frac{7}{9}$ to 1
13 games running,	are	8191 to 1
12 out of 13,	are	584 to 1
11 out of 13,	are	88 to 1
10 out of 13,	are	$20\frac{2}{3}$ to 1
9 out of 13,	are	$6\frac{1}{2}$ to 1
8 out of 13,	are	near 9 to 4, or $2\frac{1}{4}$ to 1
12 games running,	are	4095 to 1
11 out of 12,	are	314 to 1
10 out of 12,	are	$50\frac{67}{79}$, or near 51 to 1
9 out of 12,	are	$12\frac{2}{3}$ to 1
8 out of 12,	are	$4\frac{3}{20}$, or near $4\frac{1}{7}$ to 1

7 out of 12, -	are	near 8 to 5, or near $1\frac{3}{5}$ to 1
even games in 12,	are -	near $3\frac{10}{23}$, or near $3\frac{4}{9}$ to 1
11 games running,	are	- 2047 to 1
10 out of 11, -	are	- - 169 to 1
9 out of 11, -	are	- - $29\frac{1}{2}$ to 1
8 out of 11, -	are	- $7\frac{24}{29}$, or near $7\frac{5}{6}$ to 1
7 out of 11, -	are	near 13 to 5, or $2\frac{3}{5}$ to 1
10 games running,	are	- 1023 to 1
9 out of 10, -	are	- - 92 to 1
8 out of 10,	are	- - $17\frac{1}{4}$ to 1
7 out of 10, -	are -	53 to 11, or near $4\frac{4}{5}$ to 1
6 out of 10, -	are -	near 13 to 8, or $1\frac{5}{8}$ to 1
even games in 10,	are	- near $3\frac{1}{16}$ to 1
9 games running,	are	- - 511 to 1
8 out of 9, -	are	- - 50 to 1
7 out of 9, -	are	$10\frac{3}{23}$, or near $10\frac{1}{8}$ to 1
6 out of 9, -	are -	$2\frac{61}{65}$, or near 3 to 1
8 games running, -	are	- 255 to 1
7 out of 8, -	are	- - $27\frac{4}{9}$ to 1
6 out of 8, -	are -	$5\frac{34}{37}$, or near 6 to 1
5 out of 8, -	are	near 7 to 4, or $1\frac{3}{4}$ to 1
even games in 8,	are	near 8 to 3, or $2\frac{2}{3}$ to 1
7 games running,	are	- 127 to 1
6 out of 7 -	are	- - 15 to 1
5 out of 7, -	are	near 17 to 5, or $3\frac{2}{5}$ to 1
6 games running,	are	- - 63 to 1
5 out of 6, -	are	- - $8\frac{1}{7}$ to 1
4 out of 6, -	are	21 to 11, or near 2 to 1
even games in 6,	are	11 to 5, or $2\frac{1}{5}$ to 1
5 games running,	are -	- 31 to 1
4 out of 5, -	are	near 13 to 3, or $4\frac{1}{3}$ to 1

4 games running,	are	15 to 1
3 out of 4,	are	11 to 5, or $2\frac{1}{5}$ to 1
even games in 4,	are	5 to 3, or $1\frac{2}{3}$ to 1
3 games running,	are	7 to 1
2 games running,	are	3 to 1

II. *When the Odds or Chances on each game, are 6 to 5.*

Against winning

10 games running,	the odds are	427 to 1
9 out of 10,	are	44 to 1
8 out of 10,	are	$9\frac{1}{2}$ to 1
7 out of 10,	are	$2\frac{9}{10}$, or near 3 to 1
6 out of 10,	are	nearly equal, or $1\frac{1}{17}$ to 1
even games in 10,	are	$3\frac{1}{5}$ to 1
9 games running,	are	232 to 1
8 out of 9,	are	$26\frac{1}{2}$ to 1
7 out of 9,	are	very near 6 to 1
6 out of 9,	are	near 15 to 8, or $1\frac{7}{8}$ to 1
5 out of 9,	are	near 14 to 9, or $1\frac{5}{9}$ to 1
8 games running,	are	126 to 1
7 out of 8,	are	$15\frac{1}{2}$ to 1
6 out of 8,	are	15 to 4, or $3\frac{3}{4}$ to 1
5 out of 8,	are	11 to 10, or $1\frac{1}{10}$ to 1
even games in 8,	are	14 to 5, or $2\frac{4}{5}$ to 1
7 games running,	are	68 to 1
6 out of 7,	are	$9\frac{1}{5}$ to 1
5 out of 7,	are	9 to 4, or $2\frac{1}{4}$ to 1
4 out of 7,	are	near 3 to 2, or $1\frac{1}{2}$ to 1
6 games running,	are	$36\frac{45}{46}$, or near 37 to 1
5 out of 6,	are	$5\frac{1}{4}$ to 1

4 out of 6,	are	13 to 10, or $1\frac{3}{10}$ to 1
even games in 6,	are	9 to 4, or $2\frac{1}{4}$ to 1
5 games running,	are	$19\frac{2}{3}$ to 1
4 out of 5,	are	rather better than 3 to 1
3 out of 5,	are	near 7 to 5, or $1\frac{2}{5}$ to 1
4 games running,	are	$10\frac{1}{4}$ to 1
3 out of 4,	are	near 8 to 5, or $1\frac{3}{5}$ to 1
even games in 4,	are	near 5 to 3, or $1\frac{2}{3}$ to 1
3 games running,	are	near 31 to 6, or $5\frac{1}{6}$ to 1
2 out of 3,	are	near 10 to 7, or $1\frac{3}{7}$ to 1
2 games running,	are	near 7 to 3, or $2\frac{1}{3}$ to 1
1 game each, out of 2,	are	near 61 to 60, or $1\frac{1}{60}$ to 1

III. *When the Odds or Chances on each game, are 5 to 6.*

Against winning

10 games running,	are	2654 to 1
9 out of 10,	are	203 to 1
8 out of 10,	are	33 to 1
7 out of 10,	are	$8\frac{1}{4}$ to 1
6 out of 10,	are	18 to 7, or $2\frac{4}{7}$ to 1
9 games running,	are	1206 to 1
8 out of 9,	are	101 to 1
7 out of 9,	are	$17\frac{1}{4}$ to 1
6 out of 9,	are	$4\frac{1}{2}$ to 1
8 games running,	are	547 to 1
7 out of 8,	are	$50\frac{1}{4}$ to 1
6 out of 8,	are	$9\frac{1}{4}$ to 1
5 out of 8,	are	near 19 to 7, or $2\frac{5}{7}$ to 1

7 games running,		are	-	248 to 1
6 out of 7,	-	are	-	$25\frac{1}{2}$ to 1
5 out of 7,	-	are	-	$5\frac{1}{4}$ to 1
6 games running,		are	-	112 to 1
5 out of 6,	-	are	-	$12\frac{3}{4}$ to 1
4 out of 6,	-	are	near 14 to 5, or	$2\frac{4}{5}$ to 1
5 games running,		are	-	$50\frac{1}{2}$ to 1
4 out of 5,	-	are	-	$6\frac{1}{3}$ to 1
4 games running,		are	-	$22\frac{1}{3}$ to 1
3 out of 4,	-	are	very nearly	$3\frac{1}{25}$ to 1
3 games running,		are	near	$9\frac{2}{3}$ to 1
2 games running,		are	near	$3\frac{4}{5}$ to 1

IV. *When the Odds or Chances on each game, are* 5 *to* 4.

Against winning

10 games running,		are	-	356 to 1
9 out of 10,	-	are	-	$38\frac{1}{2}$ to 1
8 out of 10,	-	are	-	$8\frac{1}{3}$ to 1
7 out of 10,	-	are	-	$2\frac{1}{2}$ to 1
6 out of 10,	-	are	14 to 13, or	$1\frac{1}{13}$ to 1
even games in 10,		are	13 to 4, or	$3\frac{1}{4}$ to 1
9 games running,		are	-	197 to 1
8 out of 9,	-	are	-	24 to 1
7 out of 9,	-	are	-	$5\frac{1}{3}$ to 1
6 out of 9,	-	are	near 5 to 3, or	$1\frac{2}{3}$ to 1
5 out of 9,	-	are	12 to 7, or	$1\frac{5}{7}$ to 1
8 games running,		are	-	109 to 1
7 out of 8,	-	are	-	$13\frac{3}{4}$ to 1
6 out of 8,	-	are	-	$3\frac{1}{3}$ to 1
5 out of 8,	-	are	near 22 to 21, or	$1\frac{1}{21}$ to 1
even games in 8,		are	near 17 to 6, or	$2\frac{5}{6}$ to 1

7 games running,	are	60 to 1
6 out of 7,	are	$8\frac{1}{4}$ to 1
5 out of 7,	are	2 to 1
4 out of 7,	are	near 5 to 3, or $1\frac{2}{3}$ to 1
6 games running,	are	33 to 1
5 out of 6,	are	near 39 to 8, or $4\frac{7}{8}$ to 1
4 out of 6,	are	near 6 to 5, or $1\frac{1}{5}$ to 1
even games in 6,	are	near 11 to 5, or $2\frac{1}{5}$ to 1
5 games running,	are	$17\frac{8}{9}$ to 1
4 out of 5,	are	near 14 to 5, or $2\frac{4}{5}$ to 1
3 out of 5,	are	near 3 to 2, or $1\frac{1}{2}$ to 1
4 games running,	are	very near $9\frac{1}{3}$ to 1
3 out of 4,	are	near 3 to 2, or $1\frac{1}{2}$ to 1
even games in 4,	are	near 17 to 10, or $1\frac{7}{10}$ to 1
3 games running,	are	near $4\frac{5}{6}$ or near 5 to 1
2 out of 3,	are	near 4 to 3, or $1\frac{1}{3}$ to 1
2 games running,	are	56 to 25, or near $2\frac{1}{5}$ to 1
1 game each out of 2,	are	41 to 40, or $1\frac{1}{40}$ to 1

V. *When the Odds or Chances on each game, are 4 to 5.*

Against winning

10 games running,	are	3324 to 1
9 out of 10,	are	245 to 1
8 out of 10,	are	$38\frac{1}{2}$ to 1
7 out of 10,	are	$9\frac{1}{3}$ to 1
6 out of 10,	are	3 to 1
9 games running,	are	1476 to 1
8 out of 9,	are	119 to 1
7 out of 9,	are	$20\frac{1}{3}$ to 1
6 out of 9,	are	$5\frac{1}{3}$ to 1

8 games running,		are	- 655 to 1
7 out of 8,	-	are	- 58 to 1
6 out of 8,	-	are	- near 11 to 1
5 out of 8,	-	are	- 3 to 1
7 games running,		are	- 290 to 1
out of 7,	-	are	$28\frac{3}{4}$ to 1
5 out of 7,	-	are	- $5\frac{6}{7}$, or near 6 to 1
6 games running,		are	- 128 to 1
5 out of 6,	-	are	- near $14\frac{1}{4}$ to 1
4 out of 6,	-	are	- 3 to 1
5 games running,		are	- $56\frac{1}{2}$ to 1
4 out of 5,	-	are	near $6\frac{71}{74}$, or 7 to 1
4 games running,		are	- $24\frac{1}{2}$ to 1
3 out of 4,	-	are	- 3 to 1
3 games running,		are	$10\frac{25}{64}$, or near $10\frac{2}{5}$ to 1
2 games running,		are	65 to 16, or $4\frac{1}{16}$ to 1

VI. *When the Odds or Chances on each game, are 6 to 4.*

Against winning

6 times running,		are	- near $20\frac{3}{7}$ to 1
5 out of 6,	-	are	- near $3\frac{2}{7}$ to 1
4 out of 6,	-	are 1601 to 1424, or near $1\frac{1}{8}$ to 1	
even games in 6,		are	- near $2\frac{5}{8}$ to 1
5 times running,		are	- near $11\frac{5}{6}$ to 1
4 out of 5,	-	are	very near 2 to 1
3 out of 5,	-	are	near 31 to 15, or $2\frac{1}{15}$ to 1
4 times running,		are	- near $6\frac{5}{7}$ to 1
3 out of 4,	-	are	328 to 297, or 11 to 10
even games in 4,		are	409 to 216, or 15 to 8

3 times running,	are -	$3\frac{17}{27}$, or near $3\frac{2}{3}$ to 1
2 out of 3,	are	81 to 44, or near $1\frac{5}{6}$ to 1
2 times running,	are	16 to 9, or $1\frac{7}{9}$ to 1
1 game only in 2,	are	13 to 12, or $1\frac{1}{12}$ to 1

VII. *When the Chances on each game, are 4 to 6.*

Against winning

6 times running,	are -	243 to 1
5 out of 6, -	are -	near $23\frac{2}{5}$ to 1
4 out of 6, -	are	near 32 to 7, or $4\frac{1}{7}$ to 1
5 times running,	are -	96 to 1
4 out of 5, -	are -	near $10\frac{1}{2}$ to 1
4 times running,	are -	$38\frac{1}{16}$ to 1
3 out of 4, -	are -	near $4\frac{1}{2}$ to 1
3 times running,	are -	$14\frac{5}{8}$ to 1
2 times running,	are	21 to 4, or $5\frac{1}{4}$ to 1

VIII. *When the Odds on each game, are 7 to 4.*

Against winning

6 times running, -	are -	$14\frac{1}{18}$ to 1
5 out of 6, -	are	near 12 to 5, or $2\frac{2}{5}$ to 1
4 out of 6, -	are	near 109 to 67, or 5 to 3
equal times in 6, -	are	very near 3 to 1
5 times running, -	are -	near $8\frac{4}{7}$ to 1
4 out of 5, -	are	near 97 to 65, or 3 to 2
3 out of 5, -	are	near 29 to 10, or near 3 to 1
4 times running,	are	51 to 10, or near 5 to 1
3 out of 4, -	are	near 19 to 9, or $2\frac{1}{9}$ to 1

3 times running,	are	near 23 to 8, or near 3 to 1
2 out of 3,	are	7 to 3, or $2\frac{1}{3}$ to 1
2 times running,	are	72 to 49, or 1[illegible] to 1
1 in 2, or even in 2,	are	65 to 56, or 7 to 6

IX. *When the Chance on each game, is 4 to 7.*

Against winning

6 times running,	are	431 to 1
5 out of 6,	are	$36\frac{1}{2}$ to 1
4 out of 6,	are	$6\frac{1}{2}$ to 1
5 times running,	are	156 to 1
4 out of 5,	are	$15\frac{1}{8}$ to 1
4 times running,	are	56 to 1
3 out of 4,	are	$6\frac{1}{7}$ to 1
3 times running,	are	$19\frac{4}{5}$ to 1
2 times running,	are	$6\frac{9}{16}$ to 1

X. *When the Odds on each game, are 2 to 1.*

Against winning

6 times running,	are	near $10\frac{1}{3}$ to 1
5 out of 6,	are	473 to 256, or 11 to 6
4 out of 6,	are	near 17 to 8, or $2\frac{1}{8}$ to 1
3 out of 6, or even	are	near 7 to 2, or $3\frac{1}{2}$ to 1
5 times running,	are	near 33 to 5, or $6\frac{3}{5}$ to 1
4 out of 5,	are	131 to 112, or near 7 to 6
3 out of 5,	are	near 34 to 9, or $3\frac{7}{9}$ to 1
4 times running,	are	65 to 16, or near 4 to 1
3 out of 4,	are	16 to 11, or $1\frac{5}{11}$ to 1
2 out of 4, or even in 4,	are	19 to 8, or $2\frac{3}{8}$ to 1

3 times running,	are	-	19 to 7, or $2\frac{5}{7}$ to 1
2 out of 3,	are		20 to 7, or near 3 to 1
2 times running,	are	-	5 to 4, or $1\frac{1}{4}$ to 1
1 in 2, or even in 2,	are	-	5 to 4, or $1\frac{1}{4}$ to 1

XI. *When the Chance on each game is $\frac{1}{2}$, or 1 to 2.*

Against winning

6 times running	are	-	728 to 1
5 out of 6,	are	- -	55 to 1
4 out of 6,	are		$8\frac{12}{13}$, or near 9 to 1
5 times running,	are	-	242 to 1
4 out of 5,	are		$21\frac{1}{11}$, or near 21 to 1
4 times running,	are	- -	80 to 1
3 out of 4,	are	-	8 to 1
3 times running,	are	- -	26 to 1
2 times running,	are	- -	8 to 1

TABLE OF MEASURES.

WE shall conclude this volume with the following table of itinerant distances, or lineal measures, which have been used by different people; all expressed in numbers of English feet.

In	Feet
ARABIA.—The mile, about	6929
CHINA.—The present li	1885
The pu, equal to 10 lis,	18857
DENMARK.—The mile	25123
ENGLAND.—The mile	5280
EGYPT.—The SCHÆNE	19421
FRANCE.—The mile	6392
The small league, of 30 to a degree,	12159

In		Feet
	The mean league, of 25 to a degree,	14594
	The great or marine league, 20 to a deg.	18238
GAUL, *(ancient.)*	The leug, or league	7249
GERMANY.	The league, or rast	14498
	The mile, 12½ to a degree	28995
	Ditto, 15 to a degree	24292
GREECE *(ancient.)*	The Olympic stadium	604
	A smaller stadium	482
	The least stadium	322
INDIA.	The little coff	8579
	The great coff	9857
	The gan (Malabar coast)	38356
	The nari, or nali, (ditto)	5753
IRELAND.	The mile	6724
ITALY.	The Roman mile	4909
	The Lombard mile	5425
	The Venetian mile	6341
JUDEA.	The rast, or stade	486
	The berath, or mile	3640
PERSIA.	The parasang, or farsang	14499
POLAND.	The league	18223
ROMAN EMPIRE.	The military mile	4833
RUSSIA.	The ancient werst	4193
	The modern werst	3497
SCOTLAND.	The mile	7332
SPAIN.	The league (legal)	13724
	The common league	20846
SWEDEN.	The mile	35050
TURKY.	The agash	16211

FINIS.

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